Received: 3 October 2022 | Revised: 9 November 2023 | Accepted: 13 December 2023

DOI: 10.1002/rnc.7169

RESEARCH ARTICLE

WILEY

Connectivity-preserving consensus: An adaptive event-triggered strategy

Tiantian Guo[1,2] | **Yungang Liu**[1]

[1]School of Control Science and Engineering, Shandong University, Jinan, People's Republic of China

[2]Department of Mathematics, School of Sciences, Hangzhou Dianzi University, Hangzhou, People's Republic of China

Correspondence
Yungang Liu, School of Control Science and Engineering, Shandong University, Jinan 250061, People's Republic of China.
Email: lygfr@sdu.edu.cn

Funding information
National Natural Science Foundation of China, Grant/Award Numbers: 62033007, 61821004; Taishan Scholars Climbing Program of Shandong Province; Major Fundamental Research Program of Shandong Province.

Summary

The connectivity of communication graph is indispensable for consensus of multi-agent systems (MASs), which in many applications (e.g., wireless sensor) depends on the relative distances between agents. But, in the adaptive setting particularly with system nonlinearities and event-triggered communication, it is rather difficult to enforce the relative distances within the limited range for the connectivity. This paper focuses on developing an adaptive event-triggered control strategy with connectivity preservation in the context of nonidentical unknown control coefficients and heterogeneous nonlinearities coupling with parameter uncertainties. First, a group of potential functions are introduced acting as control barrier functions to constrain the relative distances between agents within the limited range for all time. Also, two dynamic gains are specialized for each agent to handle the system uncertainties, system nonlinearities and negative effect of the execution error. Then, an adaptive event-triggered protocol is designed for each agent such that the connectivity-preserving consensus of MASs is achieved and Zeno behavior is excluded. Moreover, an extended study is conducted on a leader-following scenario. Two simulation examples illustrate the effectiveness of the proposed event-triggered control strategy.

KEYWORDS

adaptive event-triggered control, connectivity-preserving consensus, dynamic gains, nonlinear multi-agent systems, parameter uncertainties, unknown control coefficients

1 | INTRODUCTION

Consensus of MASs has been extensively studied for which an indispensable condition is the connectivity of communication graph.[1–12] In many real-life MASs, such as mobile wireless sensors[13] and multiple aircraft,[14] the connectivity essentially depends on the relative distances between agents. If the distances exceed a certain range, the connectivity will no longer preserve and thus consensus can not be achieved. A lot of results[15–26] have been obtained on connectivity-preserving consensus of MASs, while only a few[21,24] allow for uncertainties and the control coefficients therein are known. Actually, in the adaptive setting, enforcing the relative distances within the limited range is difficult for the connectivity.

With the increasing popularity of networked control and distributed architecture, it is necessary to reduce the occupation of shared resources as much as possible. Event-triggered control has obvious advantages in saving communication and computation resources over time-triggered one, which has been experimentally validated in works.[27,28] Concerning the topic, many results have been gained (see e.g., References 29–34). Works in References 29,30 and 31 respectively addressed the single-integrator MASs and double-integrator ones. As extensions of these three preceding works, work[32] studied general linear MASs, while work[33] admitted external disturbances for such general systems. Work[34] further allowed unknown control coefficients and system nonlinearities coupling with parameter uncertainties. Nevertheless, works[29–34] didn't involve connectivity preservation. For uncertain nonlinear MASs, event-triggered consensus problem with connectivity preservation becomes rather more challenging partly because the sampling/execution error is difficult to be accurately bounded. Despite some progress (see e.g., References 35–41), there has been no any adaptive event-triggered control strategy to handle nonidentical unknown control coefficients and heterogeneous nonlinearities coupling with parameter uncertainties. Therefore, it is urgent to pursue a powerful design strategy where suitable compensation and connectivity-preserving mechanisms should be introduced and an event-triggering mechanism is properly integrated with the previous two mechanisms to achieve connectivity-preserving consensus.

This paper aims to develop an adaptive event-triggered strategy to achieve connectivity-preserving consensus for a class of uncertain nonlinear MASs. To this end, a group of potential functions acting as control barrier functions (see e.g., References 15,18,19,24,42) are adopted. When the relative distances between any two neighboring agents approach communication radius, the corresponding potential functions would tend to infinity, which enables the control protocols to ensure the relative distances within the radius for all time. Notably, the weight coefficients induced from these potential functions (i.e., (3) below) are strictly increasing with respect to the relative distances. By virtue of the increasing property, the boundedness of the weight coefficients can be obtained once the relative distances are bounded, which is helpful in deriving the boundedness of all the closed-loop signals (see the proof of Proposition 2 below). Also, two dynamic gains are introduced for each agent to compensate the system uncertainties, suppress the nonlinearities and overcome the negative effect of the execution error. Then, an adaptive event-triggered control protocol is designed for each agent such that the connectivity of initial communication graph is maintained for all time and the consensus of MASs is achieved. Particularly, a positive lower bound for the inter-execution intervals is ensured to exclude Zeno behavior. Moreover, the developed event-triggered strategy is also extended to a leader-following scenario.

In detail, the main contributions of this paper consist of two aspects: (i) The studied systems in the connectivity-preserving framework are much more general than the existing works.[15,17–19,21] Nonidentical unknown control coefficients and heterogeneous nonlinearities coupling with parameter uncertainties are allowed simultaneously, while works[15,17–19,21] excluded unknown control coefficients and particularly works[15,17] only investigated linear MASs. (ii) The proposed adaptive event-triggered control strategy not only achieves asymptotic consensus of MASs but also guarantees the connectivity preservation. This is because both dynamic gains and potential functions are incorporated into the event-triggering mechanism. Whereas works[34,43,44] merely introduced dynamic gains and thus didn't achieve connectivity preservation.

The rest of this paper is organized as follows. Section 2 gives preliminaries and problem formulation. Section 3 presents an adaptive event-triggered connectivity-preserving consensus protocol, and the main results are summarized in Section 4. Section 5 extends the adaptive event-triggered connectivity-preserving strategy to a leader-following scenario. Two simulation examples and concluding remarks are provided in Sections 6 and 7, respectively.

2 | PRELIMINARIES AND PROBLEM FORMULATION

2.1 | Notations and graph theory

Let $\mathbf{1_N}$ denote the N-dimensional column vector whose elements are all equal to 1, I_N the identity matrix with dimension N, and $\mathbf{diag}\{x_1, \dots, x_N\}$ the N-dimensional diagonal matrix with diagonal elements x_i's. For a real number x, $\mathbf{sign}(x)$ denotes its sign function, that is, $\mathbf{sign}(x) = 1$ if $x > 0$, $\mathbf{sign}(x) = 0$ if $x = 0$, and $\mathbf{sign}(x) = -1$ if $x < 0$. For a scalar function $f(s)$, we use $D^+f(s)$ to denote the upper Dini derivative of $f(s)$, and denote it by $f(\cdot)$ or f for convenience when no confusion occurs.

For a weighted undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{W})$ with N nodes, $\mathcal{V} = \{1, \dots, N\}$ denotes the set of nodes, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ represents the set of edges and $\mathcal{W} = (\omega_{ij})_{N\times N}$ stands for the weighted adjacency matrix. $(i,j) \in \mathcal{E}$ means that there exists an edge between nodes i and j and also implies that node j is the neighbor of node i and vice versa. If $(i,j) \in \mathcal{E}$, then $\omega_{ij} > 0$, otherwise, $\omega_{ij} = 0$. $\mathcal{N}_i = \{j \in \mathcal{V} : (j,i) \in \mathcal{E}\}$ is used to represent all the neighbors of node i. m is used to denote the number of edges in graph $\mathcal{G}$ and these edges are labeled as $e_1, \dots, e_m$. Let $W = \mathbf{diag}\{w(e_1), \dots, w(e_m)\}$, where $w(e_k) = \omega_{ij}$ with e_k being the label of edge (i,j). After assigning a direction to each edge, the $n \times m$ incidence matrix $D = (d_{ij})_{n\times m}$ is defined as: When node i is the tail of edge e_j, $d_{ij} = -1$; when node i is the head of edge e_j, $d_{ij} = 1$; otherwise, $d_{ij} = 0$.

$\mathcal{L}_\omega$ denotes the edge Laplacian matrix and $\mathcal{L}_\omega = DWD^{\mathrm{T}}$. Assume that graph $\mathcal{G}$ does not contain self-loops. $\mathcal{L}$ denotes the Laplacian matrix of graph $\mathcal{G}$ and its elements are defined as follows: $l_{ij} = -\omega_{ij}$ if $i \neq j$; $l_{ij} = \sum_{j\neq i}^{N} \omega_{ij}$ if $i = j$. A path of length k connecting nodes i and j is an ordered sequence of edges: $(i_0, i_1), (i_1, i_2), \dots, (i_{k-1}, i_k)$, where $i_0 = i$, $i_k = j$, $(i_m, i_{m+1}) \in \mathcal{E}$, $0 \leq m \leq k-1$. A graph $\mathcal{G}$ is connected if there exists a path between any two different nodes.

The following lemma is presented, which will be used frequently in the later development.

Lemma 1 (38). *Let $K_N = I_N - \frac{1}{N}\mathbf{1_N}\mathbf{1_N}^{\mathrm{T}}$ and suppose $\mathcal{G}$ is connected. Then $K_N\mathcal{L}_\omega = \mathcal{L}_\omega$ and $0 \leq \lambda_2 K_N \leq DD^{\mathrm{T}}$, where λ_2 is the minimum positive eigenvalue for matrix DD^{T}.*

2.2 | Systems and control objective

This paper studies the connectivity-preserving consensus via adaptive event-triggered control for a class of uncertain nonlinear MASs consisting of N agents. The dynamics of agent i, $i \in \mathcal{V}$ is in the form below:

$$\begin{cases} \dot{x}_i = v_i, \\ \dot{v}_i = g_i u_i + \phi_i^{\mathrm{T}}(v_i)\theta_i, \end{cases} \tag{1}$$

where $x_i \in \mathbf{R}$ and $v_i \in \mathbf{R}$ are the system states; $u_i \in \mathbf{R}$ is the control input; $\phi_i(v_i) \in \mathbf{R}^{q_i}$ is a known continuously differentiable function; g_i, called the *control coefficient*, is an unknown nonzero constant with a known sign; $\theta_i \in \mathbf{R}^{q_i}$ is an unknown constant parameter vector. Actually, various actual plants, such as spacecraft[45] and surface vessels,[46] can be covered by system (1).

In this paper, since the communication range of each agent is limited, edge between agents i and j is formed if $|x_i - x_j| < r$, that is, $\mathcal{E} = \{(i,j) \in \mathcal{V} \times \mathcal{V} : |x_i - x_j| < r\}$, where r is the communication radius. Note that we do not consider the generation of new communication link.

For consensus of MASs, its process contains transient and steady-state stages. In the steady-state stage, the relative distances between any two neighboring or non-neighboring agents are maintained within the limited range. Whereas, in the transient stage, despite small initial relative distances, due to severe uncertainties and nonlinearities, the relative distances could exceed the radius r as multi-agents evolve. This requires control protocols to put special effort into keeping the relative distances within the radius r.

In detail, the control objective for system (1) is to develop an adaptive event-triggered strategy such that the consensus of MASs and connectivity preservation are achieved and sampling/execution is reduced. To achieve the desired objective, the following assumption is imposed on an initial communication graph, which is clearly basic and common in the related works (see e.g., References 15–25 and references therein).

Assumption 1. The initial communication graph among N agents is connected, that is, $|x_i(0) - x_j(0)| < r$ for any two neighboring agents i and j.

Based on Assumption 1, we try to design a suitable control protocol for each agent to ensure that the relative distances between any two initially neighboring agents are maintained within the radius r for all time, thereby making the connectivity-preserving consensus of system (1) possible. Notably, for system (1) with Assumption 1, its variants have been investigated.[15,17–19,21] To be specific, work[15] considered agents with single-integrator dynamics. Subsequently, work[17] was concerned with double-integrator dynamics. As extensions, work[18] studied second-order MASs whose nonlinearities are homogeneous and satisfy a Lipschitz-like condition, while work[19] took heterogeneous nonlinearities into account and the differences between any two agents' nonlinearities therein are bounded. Work[21] further considered heterogeneous nonlinearities coupling with parameter uncertainties. Nevertheless, in the works,[15,17–19,21] the unknown control coefficients and particularly event-triggered communication are excluded.

3 | ADAPTIVE EVENT-TRIGGERED CONTROL DESIGN

This section pursues an adaptive event-triggered protocol for system (1) under Assumption 1 to achieve connectivity-preserving consensus. First, a group of potential functions are given to constrain the relative distances between agents within the radius r. Moreover, two dynamic gains are introduced for each agent to handle the system uncertainties and the nonlinearities while overcoming the negative effect of the execution error. Particularly, the proposed event-triggering mechanism is in a fully distributed form, that is, the triggering mechanisms of any two agents are mutually independent.

In what follows, a group of potential functions are constructed for any $i,j \in \mathcal{V}$:

$$P_{ij}(|x_i - x_j|) = \begin{cases} \frac{(x_i-x_j)^2}{r-|x_i-x_j|}, & (i,j) \in \mathcal{E}, \\ 0, & (i,j) \notin \mathcal{E}. \end{cases} \tag{2}$$

From this, we derive that $\frac{\partial P_{ij}(|x_i-x_j|)}{\partial x_i} = \frac{2r-|x_i-x_j|}{(r-|x_i-x_j|)^2}(x_i - x_j)$ when $(i,j) \in \mathcal{E}$, $\frac{\partial P_{ij}(|x_i-x_j|)}{\partial x_i} = 0$ otherwise. For convenience, denote by $\omega_{ij}(|x_i - x_j|)$ the weight coefficient of the partial derivative of $P_{ij}(|x_i - x_j|)$ with respect to x_i. Then, we have

$$\omega_{ij}(|x_i - x_j|) = \begin{cases} \frac{2r-|x_i-x_j|}{(r-|x_i-x_j|)^2}, & (i,j) \in \mathcal{E}, \\ 0, & (i,j) \notin \mathcal{E}. \end{cases} \tag{3}$$

From (2) and (3), we know that $P_{ij}(|x_i - x_j|)$ and $\omega_{ij}(|x_i - x_j|)$ enjoy certain nice properties. Detailedly, for any $(i,j) \in \mathcal{E}$, $P_{ij}(|x_i - x_j|)$ is positive definite and radially unbounded, that is, $P_{ij}(0) = 0$, $P_{ij}(|x_i - x_j|) > 0$, $\forall |x_i - x_j| \neq 0$ and $\lim_{|x_i-x_j|\to r} P_{ij}(|x_i - x_j|) = +\infty$. This implies that $P_{ij}(|x_i - x_j|)$ acts as a control barrier function. Besides, $\omega_{ij}(|x_i - x_j|)$ as well as its derivatives of all orders is strictly increasing, by which the boundedness of all the closed-loop signals can be derived once the relative distances between any two neighboring agents are bounded (see the proof of Proposition 2 later). In the later development, for brevity, we denote $P_{ij}(|x_i - x_j|)$ and $\omega_{ij}(|x_i - x_j|)$ by $P_{ij}(t)$ and $\omega_{ij}(t)$ (or P_{ij} and ω_{ij}), respectively, when no confusion arises.

We then propose the adaptive event-triggered protocol of the form below for any $i \in \mathcal{V}$:

$$\begin{cases} u_i(t) = -\hat{\rho}_i\left(t_k^i\right)\left(\frac{3}{2}z_i\left(t_k^i\right) + \phi_i^{\mathrm{T}}\left(v_i\left(t_k^i\right)\right)\hat{\theta}_i\left(t_k^i\right) - D^+\alpha_i\left(t_k^i\right)\right), \quad \forall t \in \left[t_k^i,\ t_{k+1}^i\right), \\ \alpha_i(t) = -\sum_{j\in\mathcal{N}_i} \omega_{ij}(|x_i(t) - x_j(t)|)(x_i(t) - x_j(t)), \\ \dot{\hat{\theta}}_i = \phi_i(v_i)z_i, \quad z_i = v_i - \alpha_i, \\ \dot{\hat{\rho}}_i = \mathbf{sign}(g_i)z_i\left(\frac{3}{2}z_i + \phi_i^{\mathrm{T}}(v_i)\hat{\theta}_i - D^+\alpha_i\right), \end{cases} \tag{4}$$

where t_k^i's are the sampling times of $(z_i, v_i, \hat{\theta}_i, \hat{\rho}_i, x_i - x_j)$ (or the execution times of u_i) for agent i.

In (4), $\hat{\rho}_i$ and $\hat{\theta}_i$, called the *adaptive dynamic gains* of agent i, are used to estimate $\frac{1}{g_i}$ and θ_i, respectively, where g_i and θ_i are the uncertainties in system (1). In fact, the introduction of $\hat{\rho}_i$ avoids zero division since directly estimating g_i leads to that the reciprocal of such an estimate appears in the control protocol. By leveraging the local interaction between agents, the unknown control coefficients, the nonlinearities coupling with parameter uncertainties and the negative effect of the execution error can be effectively handled by the two dynamic gains.

Moreover, for agent i, $i \in \mathcal{V}$, the execution error is defined as:

$$e_i(t) = u_i(t) + \hat{\rho}_i(t)\left(\frac{3}{2}z_i(t) + \phi_i^{\mathrm{T}}(v_i(t))\hat{\theta}_i(t) - D^+\alpha_i(t)\right), \quad \forall t \in \left[t_k^i,\ t_{k+1}^i\right). \tag{5}$$

Based on this, t_k^i's ($t_1^i = 0$) are online generated by the following event-triggering mechanism:

$$t_k^i = \inf\left\{t > t_{k-1}^i \,\middle|\, |e_i(t)| \geq \eta_i(t)\right\}, \tag{6}$$

where "$|e_i(t)| \geq \eta_i(t)$" is referred to as the **Event**, and $\eta_i(t) > 0$ is a square integrable continuous function and satisfies $\lim_{t\to+\infty} \eta_i(t) = 0$. Actually, the threshold function $\eta_i(t)$ which fits the requirements is common, such as $\eta_i(t) = \frac{1}{\sqrt{1+t^2}}$ and $\eta_i(t) = e^{-t}$.

Remark that the introduction of $\eta_i(t)$ in (6) aims to provide a positive lower bound for the inter-execution intervals (see the proof of Theorem 1 later) and thereby guarantee the implementability of event-triggered control protocol

(4). However, we know from (6) that the initial value and the convergence speed of $\eta_i(t)$ would affect the frequency of samping/execution. To further reduce the control updating, the refined selection of function $\eta_i(t)$ is crucial. For example, under the same initial value, choosing $\eta_i(t)$ with slower convergence speed would be more conducive to resource conservation.

From (4), (5) and (6), we can see that for agent i, $i \in \mathcal{V}$, the generation of $\{t_k^i\}$ is independent of the other agents' triggering times, which means that the proposed event-triggering mechanism follows a fully distributed way and decides asynchronous samplings and executions. Moreover, the **Event** in (6) incorporates the dynamic gains (i.e., $\hat{\rho}_i$ and $\hat{\theta}_i$ in (4)), which enables the developed adaptive event-triggered strategy to handle the nonidentical unknown control coefficients, the heterogeneous nonlinearities coupling with parameter uncertainties and the negative effect of the execution error. Thus, the investigated systems are more general than those in the related works.[15,17–19,21] Besides dynamic gains, the weight coefficients (i.e., ω_{ij}'s in (3)) induced from potential functions are also incorporated the triggering mechanism, which makes the developed event-triggered strategy guarantee the connectivity preservation of initial communication graph.

The following proposition, whose proof detailedly shows the derivation process of adaptive control protocol, characterizes the integral input-to-state stable property of the closed-loop system with respect to inputs η_i's via a Lyapunov argument.

Proposition 1. *For system (1) satisfying Assumption 1, the adaptive event-triggered protocol (4) with $i \in \mathcal{V}$, under the event-triggering mechanism (6), makes the Lyapunov function $V = \frac{1}{2}\sum_{i=1}^{N}\left(\sum_{j\in\mathcal{N}_i} P_{ij} + z_i^2 + |g_i|\tilde{\rho}_i^2 + \tilde{\theta}_i^{\mathrm{T}}\tilde{\theta}_i\right)$ satisfy*

$$D^+V(t) \le -\frac{1}{2}\sum_{i=1}^{N}\left(\sum_{j\in\mathcal{N}_i}\omega_{ij}(t)\big(x_i(t)-x_j(t)\big)\right)^2 - \sum_{i=1}^{N}\frac{z_i^2(t)}{2} + \overline{g}\sum_{i=1}^{N}\eta_i^2(t), \quad t \in \left[t_k^i,\ t_{k+1}^i\right), \tag{7}$$

where $\tilde{\rho}_i = \rho_i - \hat{\rho}_i$, $\rho_i = \frac{1}{g_i}$, $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$, and $\overline{g}$ is an unknown positive constant.

Proof. We carry out the proof in two steps. First, choose $V_1 = \frac{1}{2}\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i} P_{ij}$ as the Lyapunov function candidate. Then, invoking (2) and (3), and noting $\dot{x}_i = v_i$, we derive

$$D^+V_1 = \sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i}\omega_{ij}(x_i-x_j)v_i = \sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i}\omega_{ij}(x_i-x_j)\alpha_i + \sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i}\omega_{ij}(x_i-x_j)(v_i-\alpha_i),$$

which, together with (4), yields

$$D^+V_1 = -\sum_{i=1}^{N}\left(\sum_{j\in\mathcal{N}_i}\omega_{ij}(x_i-x_j)\right)^2 + \sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i}\omega_{ij}(x_i-x_j)z_i \le -\frac{1}{2}\sum_{i=1}^{N}\left(\sum_{j\in\mathcal{N}_i}\omega_{ij}(x_i-x_j)\right)^2 + \sum_{i=1}^{N}\frac{z_i^2}{2}. \tag{8}$$

Second, let $V = V_1 + \frac{1}{2}\sum_{i=1}^{N}\left(z_i^2 + |g_i|\tilde{\rho}_i^2 + \tilde{\theta}_i^{\mathrm{T}}\tilde{\theta}_i\right)$. Then, by (1), (4) and (8), noting $\hat{\rho}_i = \frac{1}{g_i} - \tilde{\rho}_i$ and the definition of the execution error e_i in (5), we obtain

$$\begin{aligned}
D^+V \le{}& -\frac{1}{2}\sum_{i=1}^{N}\left(\sum_{j\in\mathcal{N}_i}\omega_{ij}(x_i-x_j)\right)^2 + \sum_{i=1}^{N}\frac{z_i^2}{2} + \sum_{i=1}^{N}\left(z_i\big(g_iu_i + \phi_i^{\mathrm{T}}(v_i)\theta_i - D^+\alpha_i\big) - |g_i|\tilde{\rho}_i\dot{\hat{\rho}}_i - \tilde{\theta}_i^{\mathrm{T}}\dot{\hat{\theta}}_i\right)\\
={}& -\frac{1}{2}\sum_{i=1}^{N}\left(\sum_{j\in\mathcal{N}_i}\omega_{ij}(x_i-x_j)\right)^2 - \sum_{i=1}^{N}z_i^2 - \sum_{i=1}^{N}g_i\hat{\rho}_iz_i\big(\phi_i^{\mathrm{T}}(v_i)\hat{\theta}_i - D^+\alpha_i\big)\\
&+ \sum_{i=1}^{N}\left(z_i(\phi_i^{\mathrm{T}}(v_i)\theta_i - D^+\alpha_i) - |g_i|\tilde{\rho}_i\dot{\hat{\rho}}_i - \tilde{\theta}_i^{\mathrm{T}}\dot{\hat{\theta}}_i\right) + \sum_{i=1}^{N}g_iz_ie_i
\end{aligned}$$

$$= -\frac{1}{2}\sum_{i=1}^{N}\left(\sum_{j\in\mathcal{N}_i}\omega_{ij}(x_i - x_j)\right)^2 - \sum_{i=1}^{N} z_i^2 + \sum_{i=1}^{N} g_i z_i e_i + \sum_{i=1}^{N}(\theta_i - \hat{\theta}_i)^{\mathrm{T}}\left(\phi_i(v_i)z_i - \dot{\hat{\theta}}_i\right)$$
$$+ \sum_{i=1}^{N}\left(g_i\tilde{\rho}_i z_i\left(\frac{3}{2}z_i + \phi_i^{\mathrm{T}}(v_i)\hat{\theta}_i - D^+\alpha_i\right) - |g_i|\tilde{\rho}_i\dot{\hat{\rho}}_i\right).$$

Also, from the triggering mechanism (see (6)), it follows that $|e_i(t)| \le \eta_i(t)$, $\forall t \in \left[t_k^i, t_{k+1}^i\right)$. Then, by the definitions of $\dot{\hat{\theta}}_i$ and $\dot{\hat{\rho}}_i$ in (4), and using the method of completing the square, we gain for any $t \in \left[t_k^i, t_{k+1}^i\right)$,

$$D^+V(t) \le -\frac{1}{2}\sum_{i=1}^{N}\left(\sum_{j\in\mathcal{N}_i}\omega_{ij}(t)\big(x_i(t) - x_j(t)\big)\right)^2 - \sum_{i=1}^{N}\frac{z_i^2(t)}{2} + \sum_{i=1}^{N}\frac{g_i^2\eta_i^2(t)}{2},$$

which means that (7) holds. ∎

Remark 1. It is worth pointing out that the developed adaptive event-triggered consensus strategy can ensure the collision avoidance between neighboring agents by slightly modifying the Lyapunov function V in Proposition 1. Specifically, similar to (2), a group of potential functions are introduced to avoid the collision between agents for any $i,j \in \mathcal{V} = \{1, \ldots, N\}$:

$$Q_{ij}(|x_i - x_j|) = \begin{cases} \frac{1}{(x_i-x_j)^2-(r^*)^2}, & (i,j) \in \mathcal{E}, \\ 0, & (i,j) \notin \mathcal{E}, \end{cases} \tag{9}$$

where r^* is the critical value of collision between agents and satisfies $0 < r^* < r$, with r being the communication radius defined in Subsection 2.2. From (9), we see that $\lim_{|x_i - x_j|\to r^*} Q_{ij}\left(|x_i - x_j|\right) = \infty$, which means that $Q_{ij}(|x_i - x_j|)$ acts as a control barrier function. To ensure collision avoidance and connectivity preservation simultaneously, redefine the edge set $\mathcal{E} = \{(i,j) \in \mathcal{V}\times\mathcal{V} : r^* < |x_i - x_j| < r\}$. Moreover, in Proposition 1 and its proof, the potential function P_{ij} is replaced by $Q_{ij} + P_{ij}$. As such, an inequality similar to (7) is gained, by which and adopting the performance analysis similar to that in Section 4, collision avoidance and connectivity preservation of system (1) are achieved.

4 | MAIN RESULTS

This section addresses the main results of this paper. To be specific, the connectivity preservation of initial communication graph is ensured while the consensus of MASs (1) is achieved. Besides, Zeno behavior of the closed-loop system is excluded.

From (1) and (4), we see that on any inter-execution interval $\left[t_k^i,\ t_{k+1}^i\right)$ $(i \in \mathcal{V})$, the vector field of the resulting closed-loop system is locally Lipschitz in $(x_i, v_i, \hat{\rho}_i, \hat{\theta}_i)$ and continuous in u_i. By the theorem of existence and uniqueness of solutions and the continuation theorem (see e.g., theorem 3.1 on p. 18 and theorem 2.1 on p. 17 in work[47]), differential Equations (1) and (4) with $u_i(t) \equiv u_i(t_1^i)$ have a unique solution $\left(x_i(t), v_i(t), \hat{\rho}_i(t), \hat{\theta}_i(t)\right)$ on the maximal existence interval $\left[t_1^i,\ T_{e_1}^i\right)$. When the **Event** in (6) happens on $\left[t_1^i,\ T_{e_1}^i\right)$, in the light of the event-triggering mechanism, the triggering time t_2^i is generated on $\left[t_1^i,\ T_{e_1}^i\right)$. Take $\left(x_i(t_2^i), v_i(t_2^i), \hat{\rho}_i(t_2^i), \hat{\theta}_i(t_2^i)\right)$ as a new initial value. Similarly, we can obtain that the solution of the above differential equations with $u_i(t) \equiv u_i\left(t_2^i\right)$ exists and is unique on the maximal existence interval $\left[t_2^i,\ T_{e_2}^i\right)$. This procedure would be repeated again if the next triggering time is generated. Therefore, the resulting closed-loop system has a unique solution on its maximal existence interval $[0,\ T_e)$ with $0 < T_e \le +\infty$.

Before presenting the main theorem, the following key proposition is given, which shows the role of connectivity preservation in consensus achievement. The proof of the proposition will be provided later.

Proposition 2. *If the connectivity of initial communication graph is preserved on* $[0, +\infty)$, *the consensus of MASs* (1) *is achieved, that is, for any* $i, j = 1, \ldots, N$, *and* $i \neq j$,

$$\lim_{t\to+\infty}(x_i(t) - x_j(t)) = 0, \quad \lim_{t\to+\infty}(v_i(t) - v_j(t)) = 0.$$

In the following, we are ready to give the main theorem of this paper.

Theorem 1. *For system* (1) *satisfying Assumption* 1, *the adaptive event-triggered protocol* (4) *with* $i \in \mathcal{V}$, *under the event-triggering mechanism* (6), *guarantees that the resulting closed-loop system has a unique solution on* $[0, +\infty)$ *and the connectivity of initial communication graph is preserved on* $[0, +\infty)$, *and furthermore*, $\lim_{t\to+\infty}(x_i(t) - x_j(t)) = 0$, $\lim_{t\to+\infty}(v_i(t) - v_j(t)) = 0$, $\forall i, j \in \mathcal{V}$ *and* $i \neq j$. *Besides, Zeno behaviour is excluded by ensuring* $\inf_{\{k\}}\left(t^i_{k+1} - t^i_k\right) > 0$.

Proof. Let us first show $T_e = +\infty$ by showing the boundedness of all the closed-loop signals $(x_i, v_i, u_i, \hat{\rho}_i, \hat{\theta}_i, z_i)$ $(i \in \mathcal{V})$ and the exclusion of Zeno behavior on $[0, T_e)$. Specifically, by Proposition 1, integrating both sides of (7) gets for any $t \in \left[t^i_k, t^i_{k+1}\right)$,

$$V(t) - V\left(t^i_k\right) \leq -\frac{1}{2}\sum_{i=1}^{N}\int_{t^i_k}^{t}\left(\sum_{j\in\mathcal{N}_i}\omega_{ij}(s)\left(x_i(s) - x_j(s)\right)\right)^2 \mathrm{d}s - \sum_{i=1}^{N}\int_{t^i_k}^{t}\frac{z_i^2(s)}{2}\mathrm{d}s + \overline{g}\int_{t^i_k}^{t}\sum_{i=1}^{N}\eta_i^2(s)\mathrm{d}s.$$

Then, by the continuity of V at each triggering time t^i_k, and noting $t^i_1 = 0$, we have

$$V(t) - V(0) \leq -\frac{1}{2}\sum_{i=1}^{N}\int_{0}^{t}\left(\sum_{j\in\mathcal{N}_i}\omega_{ij}(s)\left(x_i(s) - x_j(s)\right)\right)^2 \mathrm{d}s - \sum_{i=1}^{N}\int_{0}^{t}\frac{z_i^2(s)}{2}\mathrm{d}s + \overline{g}\int_{0}^{t}\sum_{i=1}^{N}\eta_i^2(s)\mathrm{d}s, \quad \forall t \in [0,\ T_e). \tag{10}$$

From this, the definition of V in Proposition 1 and the square integrability of η_i's, it follows that for any $i \in \mathcal{V}$,

$$\begin{cases} \sup\limits_{t\in[0,\ T_e)}\left(|z_i(t)| + |\hat{\rho}_i(t)| + \|\hat{\theta}_i(t)\|\right) < +\infty, \\ \int_0^{T_e}\left(\left(\sum\limits_{j\in\mathcal{N}_i}\omega_{ij}(s)\left(x_i(s) - x_j(s)\right)\right)^2 + z_i^2(s)\right)\mathrm{d}s < +\infty. \end{cases} \tag{11}$$

Also, by (10), we have for any $i, j \in \mathcal{V}$,

$$P_{ij}(t) \leq 2V(t) \leq 2V(0) + 2\overline{g}\int_0^t\sum_{i=1}^{N}\eta_i^2(s)\mathrm{d}s \leq c, \quad \forall t \in [0,\ T_e),$$

with c an unknown positive constant. Based on this, and noting the definition of P_{ij} in (2), we yield for any $(i, j) \in \mathcal{E}$,

$$|x_i(t) - x_j(t)| \leq k_{ij} = -\frac{c}{2} + \sqrt{\left(\frac{c}{2}\right)^2 + rc} < r, \quad \forall t \in [0,\ T_e). \tag{12}$$

We next show the boundedness of (x_i, v_i, u_i) on $[0,\ T_e)$ $(\forall i \in \mathcal{V})$. Note that $f(l) = \frac{2r-l}{(r-l)^2}$ is an increasing function with respect to l on $[0, r)$. Then, by the definitions of ω_{ij} in (3) and α_i in (4), and invoking $|x_i - x_j| \leq k_{ij}$ in (12), we derive that on $[0,\ T_e)$, $|\alpha_i| \leq \sum_{j\in\mathcal{N}_i}\frac{(2r-k_{ij})k_{ij}}{(r-k_{ij})^2} < +\infty$. From this and $v_i = z_i + \alpha_i$ in (4), utilizing the boundedness of z_i in (11), we obtain that $v_i(t)$ is bounded on $[0,\ T_e)$. Moreover, by $\dot{x}_i = v_i$ in (1), we gain

$$\frac{dx_i^2(t)}{dt} = 2x_i(t)v_i(t) \leq x_i^2(t) + v_i^2(t),$$

from which and $v_i = z_i - \sum_{j\in\mathcal{N}_i} \omega_{ij}(x_i - x_j)$, it follows that

$$x_i^2(t) \le e^t x_i^2(0) + \int_0^t e^{t-s} v_i^2(s)\mathrm{d}s \le 2e^t \left(x_i^2(0) + \int_0^t \left(\left(\sum_{j\in\mathcal{N}_i} \omega_{ij}(s)(x_i(s) - x_j(s)) \right)^2 + z_i^2(s) \right) \mathrm{d}s \right).$$

This, in conjunction with (11), yields that $x_i(t)$ is bounded on $[0,\ T_e^i)$. Also, for any $i,j \in \mathcal{V}$, define functions $f_{ij}(l)$, $g_{ij}(l)$ and $h_{ij}(l)$ as follows:

$$f_{ij}(l) = \frac{2r - l}{(r - l)^2}, \quad g_{ij}(l) = \frac{3r - l}{(r - l)^3}, \quad h_{ij}(l) = \frac{2(4r - l)}{(r - l)^4}, \quad l \in [0, r). \tag{13}$$

Note that functions $f_{ij}(l)$, $g_{ij}(l)$ and $h_{ij}(l)$ are increasing with respect to l on $[0, r)$. Then, by (12), we obtain for any $(i,j) \in \mathcal{E}$ and $t \in [0,\ T_e)$,

$$f_{ij}(|x_i(t) - x_j(t)|) \le f_{ij}(k_{ij}), \quad g_{ij}(|x_i(t) - x_j(t)|) \le g_{ij}(k_{ij}), \quad h_{ij}(|x_i(t) - x_j(t)|) \le h_{ij}(k_{ij}). \tag{14}$$

Based on this, we next prove the boundedness of u_i on $[0,\ T_e)$. By (13) and the definition of ω_{ij} in (3), and noting $\dot{x}_i = v_i$ and $\alpha_i = -\sum_{j\in\mathcal{N}_i} \omega_{ij}(x_i - x_j)$, we derive for any $(i,j) \in \mathcal{E}$,

$$\begin{aligned} D^+\alpha_i &= -\sum_{j\in\mathcal{N}_i} \left(\dot{\omega}_{ij}(x_i - x_j) + \omega_{ij}(v_i - v_j) \right) \\ &= -\sum_{j\in\mathcal{N}_i} \left(g_{ij}(|x_i - x_j|)|x_i - x_j|(v_i - v_j) + f_{ij}(|x_i - x_j|)(v_i - v_j) \right), \end{aligned} \tag{15}$$

which, together with (14), yields the boundedness of $D^+\alpha_i$. Then, by the continuity of $\phi_i(v_i)$ and the boundedness of $(z_i, v_i, \hat{\theta}_i)$ on $[0,\ T_e^i)$, we derive that u_i is bounded on $[0,\ T_e)$.

By the aid of the boundedness of $(x_i, v_i, u_i, \hat{\rho}_i, \hat{\theta}_i, z_i)$ $(i \in \mathcal{V})$ on $[0,\ T_e)$, we now exclude Zeno behaviour. By (1), (14) and (15), we have for any $(i,j) \in \mathcal{E}$,

$$\begin{aligned} D^+(D^+\alpha_i) &= -\sum_{j\in\mathcal{N}_i} \left(\ddot{\omega}_{ij}(x_i - x_j) + 2\dot{\omega}_{ij}(v_i - v_j) + \omega_{ij}(\dot{v}_i - \dot{v}_j) \right) \\ &= -\sum_{j\in\mathcal{N}_i} \left(h_{ij}(|x_i - x_j|)(x_i - x_j)(v_i - v_j)^2 + g_{ij}(|x_i - x_j|)|x_i - x_j|(\dot{v}_i - \dot{v}_j) \right) \\ &\quad - \sum_{j\in\mathcal{N}_i} \left(2g_{ij}(|x_i - x_j|) \frac{x_i - x_j}{|x_i - x_j|}(v_i - v_j)^2 + f_{ij}(|x_i - x_j|)(\dot{v}_i - \dot{v}_j) \right), \end{aligned}$$

which, together with (1), (14), the continuity of $\phi_i(v_i)$ and the boundedness of (x_i, v_i, u_i) on $[0,\ T_e)$, yields that $D^+(D^+\alpha_i)$ is bounded on $[0,\ T_e)$.

Noting the continuous differentiability of $\phi_i(v_i)$ and the boundedness of $(x_i, v_i, u_i, \hat{\rho}_i, \hat{\theta}_i, z_i, D^+\alpha_i, D^+(D^+\alpha_i))$, and invoking (5), we obtain that there exists a positive constant M such that, for any k,

$$|D^+e_i(t)| \le M, \quad \forall t \in \left[t_k^i, t_{k+1}^i\right). \tag{16}$$

Also, according to the triggering mechanism (6), we obtain for any $i \in \mathcal{V}$,

$$|e_i(t)| \le \eta_i(t), \quad \forall t \in \left[t_k^i,\ t_{k+1}^i\right).$$

By this and (16), we gain

$$\left| D^+e_i^2(t) \right| = \left| 2e_i(t)D^+e_i(t) \right| \le 2M\eta_i(t), \quad \forall t \in \left[t_k^i,\ t_{k+1}^i\right). \tag{17}$$

Moreover, it follows from the definition of t_k^i in (6) that $e_i^2(t_k^i) = 0$ and $e_i^2\left(t_{k+1}^{i-}\right) = \eta_i^2\left(t_{k+1}^i\right)$. Then, by (17), we get

$$\eta_i^2\left(t_{k+1}^i\right) = \int_{t_k^i}^{t_{k+1}^i} D^+ e_i^2(s)\mathrm{d}s \le \int_{t_k^i}^{t_{k+1}^i} \left|D^+ e_i^2(s)\right|\mathrm{d}s \le 2M \max_{t\in\left[t_k^i, t_{k+1}^i\right)} \eta_i(t)\left(t_{k+1}^i - t_k^i\right),$$

which yields for any $i \in \mathcal{V}$,

$$\inf_{\{k\}}\left(t_{k+1}^i - t_k^i\right) \ge \frac{\eta_i^2\left(t_{k+1}^i\right)}{2M \max\limits_{t\in\left[t_k^i, t_{k+1}^i\right)} \eta_i(t)} \ge \frac{\min\limits_{t\in[0,T_e)} \eta_i^2(t)}{2M \max\limits_{t\in[0,T_e)} \eta_i(t)} > 0.$$

This means that no Zeno behavior for the closed-loop system occurs. Hence, we gain $T_e = +\infty$.

Finally, we show that the connectivity of initial communication graph is preserved on $[0, +\infty)$. By repeating the proof of (12), noting $T_e = +\infty$, we readily derive that (12) holds for any $t \in [0, +\infty)$. As a result, the connectivity preservation of MASs (1) is achieved on $[0, +\infty)$. Then, by Proposition 2, we obtain that the states of all agents reach consensus.

This completes the proof of the theorem. ∎

Proof of Proposition 2. We first prove $\lim_{t\to+\infty}(x_i(t) - x_j(t)) = 0$ for any $i, j = 1, \dots, N$, $i \ne j$. Choose Lyapunov function $U = \frac{1}{2}x^{\mathrm{T}}K_N x$, where $x = [x_1, \dots, x_N]^{\mathrm{T}}$ and K_N is the same as that in Lemma 1. Since the communication graph $\mathcal{G}$ keeps connected on $[0, +\infty)$, from Lemma 1, it follows that $K_N L_\omega = L_\omega$ and $0 \le \lambda_2 K_N \le DD^{\mathrm{T}}$, where $L_\omega = DWD^{\mathrm{T}}$, $W = \mathbf{diag}\{w(e_1), \dots, w(e_m)\}$ and $w(e_k) = \omega_{ij}$. Also, by the definition of ω_{ij} in (3), we have for any $(i,j) \in \mathcal{E}$, $\omega_{ij}(|x_i - x_j|) > \frac{2}{r - |x_i - x_j|} > \frac{2}{r}$. Then, by $\dot{x}_i = v_i$ and $v_i = z_i - \sum_{j\in\mathcal{N}_i}\omega_{ij}(x_i - x_j)$, we arrive at

$$\begin{aligned} D^+U(t) &= x^{\mathrm{T}}(t)K_N(z(t) - L_\omega x(t)), \\ &\le -x^{\mathrm{T}}(t)DWD^{\mathrm{T}}x(t) + \frac{\lambda_2}{r}x^{\mathrm{T}}(t)K_N x(t) + \frac{r}{4\lambda_2}\sum_{i=1}^N z_i^2(t) \\ &\le -\frac{2\lambda_2}{r}x^{\mathrm{T}}(t)K_N x(t) + \frac{\lambda_2}{r}x^{\mathrm{T}}(t)K_N x(t) + \frac{r}{4\lambda_2}\sum_{i=1}^N z_i^2(t) \\ &= -\frac{2\lambda_2}{r}U(t) + \frac{r}{4\lambda_2}\sum_{i=1}^N z_i^2(t), \end{aligned} \tag{18}$$

with $z = [z_1, \dots, z_N]^{\mathrm{T}}$. This, together with the definition of U and the square integrability of z_i, $i \in \mathcal{V}$ in (11) below, yields for any $i, j = 1, \dots, N$, and $i \ne j$,

$$\int_0^t |x_i(s) - x_j(s)|^2 \mathrm{d}s < +\infty. \tag{19}$$

In addition, similar to the proofs of (11) and (12), we derive that $(z_i, \rho_i, \hat{\theta}_i, x_i - x_j)$, $i = 1, \dots, N$ are bounded on $[0, +\infty)$. Thus, by (4), we obtain the boundedness of $(v_i(t), u_i(t))$, $i = 1, \dots, N$ on $[0, +\infty)$. Then, it is derived from (1) that $\dot{x}_i(t) - \dot{x}_j(t)$, $i, j = 1, \dots, N$ are also bounded on $[0, +\infty)$. Hence, by (19), invoking Barbălat's Lemma (see e.g., Reference 48), we have for any $i, j = 1, \dots, N$, and $i \ne j$,

$$\lim_{t\to+\infty}(x_i(t) - x_j(t)) = 0.$$

We next show $\lim_{t\to+\infty}(v_i(t) - v_j(t)) = 0$ for any $i, j = 1, \dots, N$, $i \ne j$. By (11), noting $v_i = z_i - \sum_{j\in\mathcal{N}_i}\omega_{ij}(x_i - x_j)$, we gain the square integrability of v_i's on $[0, +\infty)$. Also, by the boundedness of $(v_i(t), u_i(t))$, we obtain that $\dot{v}_i(t)$, $i = 1, \dots, N$ are bounded on $[0, +\infty)$. Then, with the help of Barbălat's Lemma, we conclude $\lim_{t\to+\infty} v_i(t) = 0$,

$i = 1, \ldots, N$, which immediately yields for any $i, j = 1, \ldots, N$, and $i \neq j$,

$$\lim_{t\to+\infty} (v_i(t) - v_j(t)) = 0.$$

This completes the proof. ■

5 AN EXTENSION ON LEADER-FOLLOWING MULTI-AGENT SYSTEMS

This section further addresses the adaptive event-triggered connectivity-preserving consensus for the leader-following MASs.

The leader-following MASs contain N followers and a leader. The followers' dynamics are the same as system (1) and the leader has the following dynamics:

$$\dot{x}_0 = v_0, \quad \dot{v}_0 = 0, \tag{20}$$

where $x_0 \in \mathbf{R}$ and $v_0 \in \mathbf{R}$ are the leader's states.

The communication graph among the followers and the leader is denoted by $\overline{\mathcal{G}} = (\overline{\mathcal{V}}, \overline{\mathcal{E}}, \overline{\mathcal{W}})$, where $\overline{\mathcal{V}} = \{0, 1, \ldots, N\}$, node 0 stands for the leader and $\overline{\mathcal{W}} = (\overline{\omega}_{ij})_{N\times N}$ stands for the weighted adjacency matrix. The subgraph formed by the N followers therein is undirected. Note that the leader can not obtain any information from the followers, namely, $(i, 0) \notin \overline{\mathcal{E}}$. In the graph $\overline{\mathcal{G}}$, $(0, i) \in \overline{\mathcal{E}}$ means that the i-th follower has access to the information from the leader when $|x_i - x_0| < r$. Denote $\overline{\mathcal{E}} = \{(i, j) \in \overline{\mathcal{V}} \times \mathcal{V} : |x_i - x_j| < r\}$. To achieve the connectivity-preserving consensus of the leader-following MASs consisting of (1) and (20), the following assumption is proposed for the initial communication graph:

Assumption 2. The initial communication graph contains a directed spanning tree with the leader being the root.

By the aid of Assumption 2, we present the following adaptive event-triggered protocol for the leader-following MASs (1) and (20) with $i \in \mathcal{V}$:

$$\begin{cases} u_i\left(t_k^i\right) = -\hat{\rho}_i\left(t_k^i\right)\left(\frac{3}{2}\overline{z}_i\left(t_k^i\right) + \phi_i^{\mathrm{T}}\left(v_i\left(t_k^i\right)\right)\hat{\theta}_i\left(t_k^i\right) - D^+\overline{\alpha}_i\left(t_k^i\right)\right), \quad \forall t \in \left[t_k^i,\ t_{k+1}^i\right), \\ \overline{\alpha}_i(t) = -\sum\limits_{j\in\overline{\mathcal{N}}_i} \omega_{ij}(|x_i(t) - x_j(t)|)(x_i(t) - x_j(t)), \\ \dot{\hat{\theta}}_i = \phi_i(v_i)\overline{z}_i, \quad \overline{z}_i = v_i - v_0 - \overline{\alpha}_i, \\ \dot{\hat{\rho}}_i = \mathbf{sign}(g_i)\overline{z}_i\left(\frac{3}{2}\overline{z}_i + \phi_i^{\mathrm{T}}(v_i)\hat{\theta}_i - D^+\overline{\alpha}_i\right), \end{cases} \tag{21}$$

where $\overline{\mathcal{N}}_i = \left\{j \in \overline{\mathcal{V}} : (j, i) \in \overline{\mathcal{E}}\right\}$, $\left\{t_k^i\right\}$ is generated by the same event-triggering mechanism as (6), the arguments $u_i(t)$, $\hat{\rho}_i(t)$, $z_i(t)$, $\hat{\theta}_i(t)$ and $D^+\alpha_i(t)$ in the execution error $e_i(t)$ of (6) are replaced by the corresponding ones in (21), respectively.

To facilitate the performance analysis, let $\overline{x}_i = x_i - x_0$ and $\overline{v}_i = v_i - v_0$, $i \in \mathcal{V}$. Then, it follows that $\overline{x}_i - \overline{x}_j = x_i - x_j$, $\overline{v}_i - \overline{v}_j = v_i - v_j$. It's worth noting that due to the interaction among the leader and followers, the Lyapunov function in Proposition 1 is substituted by $V = \sum_{i=1}^N P_{0i} + \frac{1}{2}\sum_{i=1}^N \left(\sum_{j\in\overline{\mathcal{N}}_i/\{0\}} P_{ij} + \overline{z}_i^2 + |g_i|\tilde{\rho}_i^2 + \tilde{\theta}_i^{\mathrm{T}}\tilde{\theta}_i\right)$, where P_{ij}'s are defined in (2), the definitions of $\tilde{\rho}_i$ and $\tilde{\theta}_i$ are the same as those in Proposition 1, $\overline{z}_i$'s are given in (21). After slight substitutions, the corresponding proofs can proceed as those of Propositions 1 and 2 and hence are not provided here.

We now state the main theorem for the leader-following MASs (1) and (20).

Theorem 2. *For the leader-following MASs (1) and (20) satisfying Assumption 2, the adaptive event-triggered protocol (21) with $i \in \mathcal{V}$, under the event-triggering mechanism described in (21) below, guarantees that the closed-loop system has a unique solution on $[0, +\infty)$ and the connectivity of the initial communication graph is preserved on $[0, +\infty)$, and furthermore, $\lim_{t\to+\infty}(x_i(t) - x_0(t)) = 0$, $\lim_{t\to+\infty}(v_i(t) - v_0(t)) = 0$, $i \in \mathcal{V}$. Besides, Zeno behaviour is excluded by ensuring $\inf_{\{k\}}\left(t_{k+1}^i - t_k^i\right) > 0$.*

Proof. The proof is analogue to that of Theorem 1 in Section 4 and is hence omitted. ■

6 | SIMULATION EXAMPLES

This section provides two examples to illustrate the effectiveness of the proposed adaptive event-triggered connectivity-preserving consensus strategy for the leaderless MASs and the leader-following one.

Example 1. Consider the uncertain nonlinear MASs of the form (1) ($N = 4$). We take $\phi_1(v_1) = \sin(v_1)v_1^2$, $\phi_2(v_2) = v_2^2$, $\phi_3(v_3) = \cos(v_3)v_3^2$, $\phi_4(v_4) = \sin(v_4)\log(1 + v_4^2)$. The initial communication graph among four agents is depicted by Figure 1, from which, we see that Assumption 1 holds for the described MASs.

It's worth pointing out that the nonlinear functions $\phi_i(v_i)$'s taken above make the control strategies in works[15,17,19] no longer feasible. Works[15,17] only concerned on the linear MASs, while work[19] required the differences between any two agents' nonlinearities to be bounded. According to the control scheme adopted in Section 3, the adaptive event-triggered control protocol is devised in the form of (4) with the event-triggering mechanism (6). Here, we pick $r = 1.5$, $\eta_1(t) = e^{-0.2t}$, $\eta_2(t) = e^{-5t}$, $\eta_3(t) = e^{-10t}$, $\eta_4(t) = e^{-6t}$.

Let $[g_1, g_2, g_3, g_4] = [1, 0.2, 1, 0.5]$, $[\theta_1, \theta_2, \theta_3, \theta_4] = [1, 0.1, 0.1, 0.1]$. Choose the initial value $[x_1(0), x_2(0), x_3(0), x_4(0)] = [-0.3, 0.5, 0.2, 0.3]$, $[v_1(0), v_2(0), v_3(0), v_4(0)] = [0.1, 1, 0.1, 0.5]$, $[\hat{\rho}_1(0), \hat{\rho}_2(0), \hat{\rho}_3(0), \hat{\rho}_4(0)] = [-0.5, 0.2, -0.6, -1]$, $[\hat{\theta}_1(0), \hat{\theta}_2(0), \hat{\theta}_3(0), \hat{\theta}_4(0)] = [-4, 0.9, -1, -0.2]$. Then, we obtain Figures 2–7 by simulation. From Figures 2 and 3, we see that system states $(x_i, v_i), i = 1, \ldots, 4$, reach consensus, and the adaptive event-triggered control protocol u_i converges to zero and the adaptive dynamic gains $(\hat{\rho}_i, \hat{\theta}_i)$ are bounded. Figures 4–6 display that for each agent, the inter-execution times are positive after 1 s. This in turn implies the exclusion of Zeno behavior for the closed-loop system. Figure 7 indicates that the distances between initial neighboring agents are always less than $r = 1.5$. As such, the effectiveness of the proposed adaptive event-triggered connectivity-preserving consensus strategy is demonstrated.

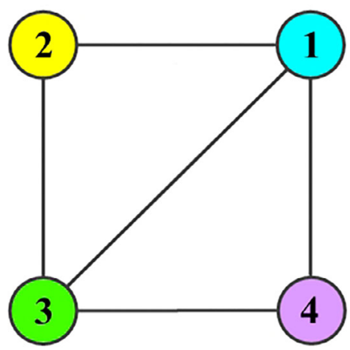

FIGURE 1 Initial communication graph among four agents in Example 1.

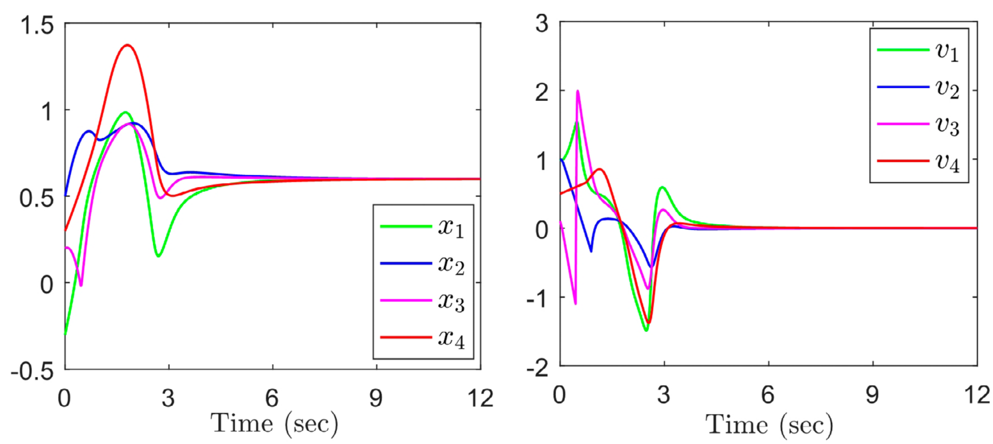

FIGURE 2 Evolution of four agents' states in Example 1.

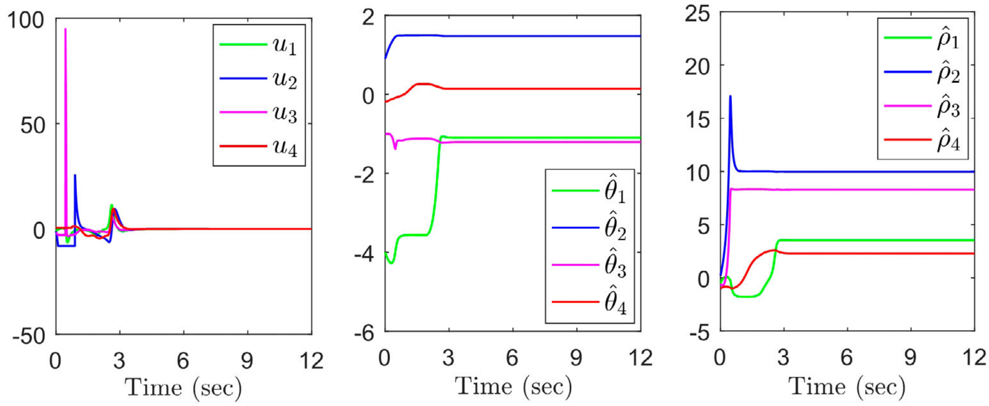

FIGURE 3 Evolution of four agents' control protocols and dynamic gains in Example 1.

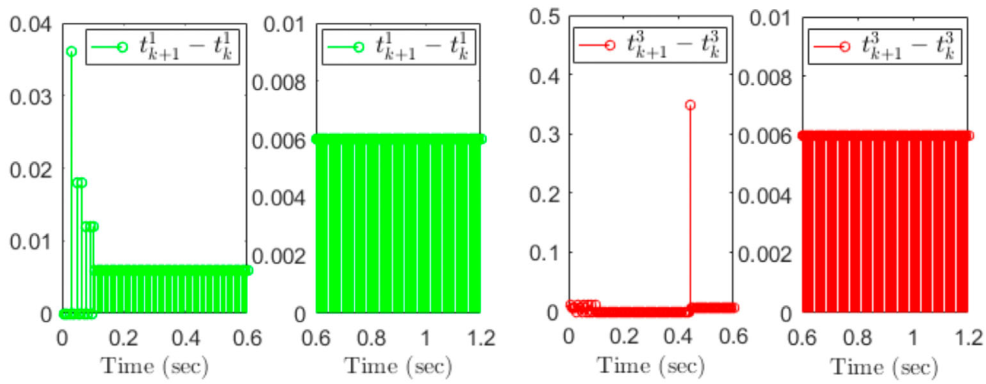

FIGURE 4 Evolution of inter-execution times of agents 1 and 2 in Example 1.

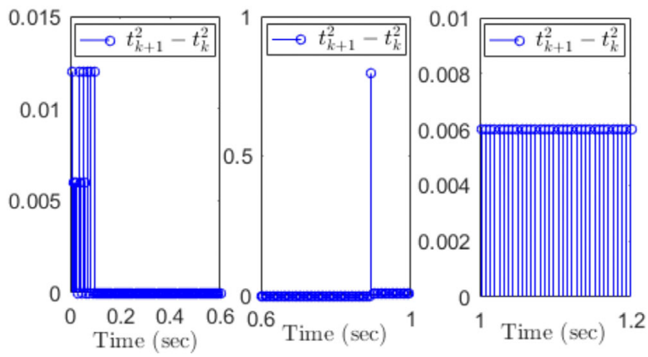

FIGURE 5 Evolution of inter-execution times of agent 3 in Example 1.

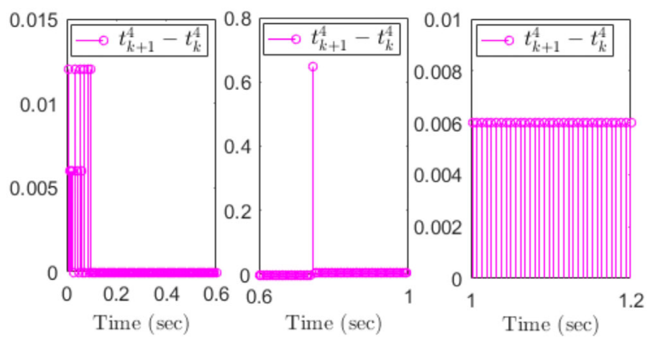

FIGURE 6 Evolution of inter-execution times of agent 4 in Example 1.

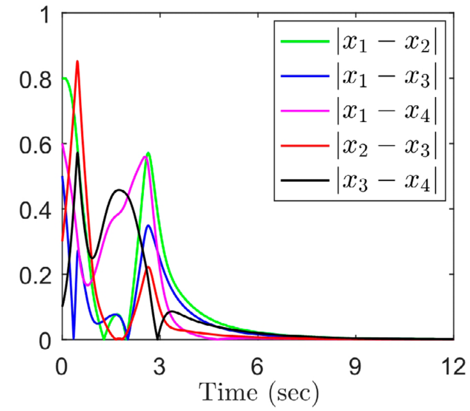

FIGURE 7 Evolution of the distances between initial neighboring agents in Example 1.

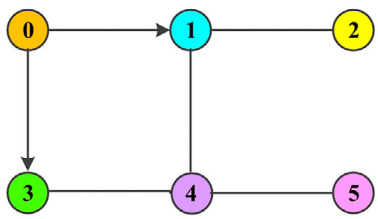

FIGURE 8 Initial communication graph among five followers and a leader in Example 2.

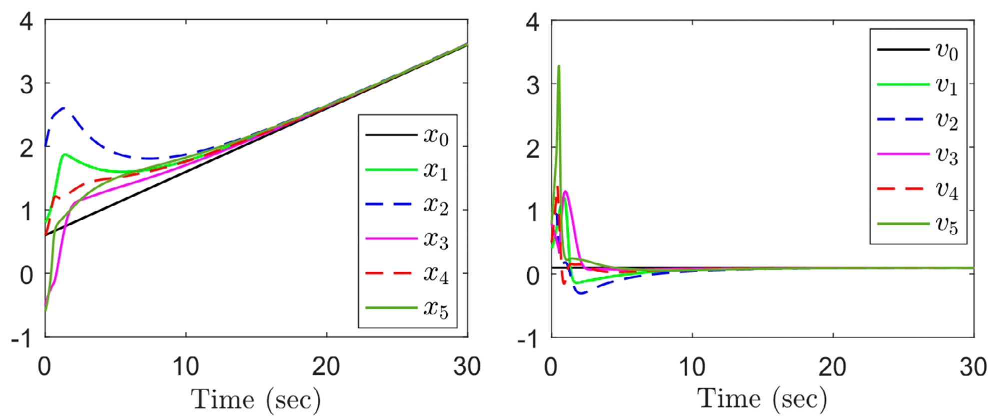

FIGURE 9 Evolution of the five followers' states and the leader' states in Example 2.

Example 2. Consider the leader-following MASs with five followers and a leader. The dynamics of the followers and the leader are of form (1) and (20), respectively. The followers' nonlinearities therein are as follows: $\phi_1(v_1) = \sin(v_1)v_1^2$, $\phi_2(v_2) = \cos(v_2)v_2^2$, $\phi_3(v_3) = \frac{v_3^2}{1+v_3^2}$, $\phi_4(v_4) = \sin(v_4)\log\left(1+v_4^2\right)$, $\phi_5(v_5) = v_5\log\left(1+v_5^2\right)$. The initial communication graph among the five followers and the leader is shown by Figure 8, from which, we see that leader 0 sends information to followers 1 and 3, but does not have access to information from the five followers.

In the light of Section 5, we adopt the adaptive event-triggered protocol of form (21). The thresholds $\eta_i(t)$'s involved in the five triggering mechanisms are chosen as $\eta_1(t) = e^{-20t}$, $\eta_2(t) = e^{-38t}$, $\eta_3(t) = e^{-26t}$, $\eta_4(t) = e^{-35t}$, $\eta_5(t) = e^{-40t}$. Also, we select the communication radius $r = 3.5$. Let $[g_1, g_2, g_3, g_4, g_5] = [-1, 0.2, 1, 0.5, -0.8]$, $[\theta_1, \theta_2, \theta_3, \theta_4, \theta_5] = [1, 0.1, 0.3, 0.1, 0.4]$. Choose the initial value $[x_0(0), x_1(0),$

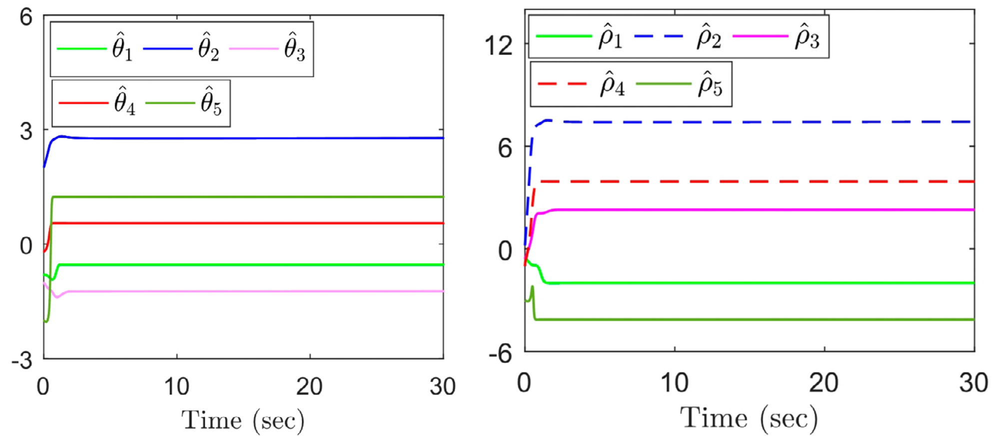

FIGURE 10 Evolution of the five followers' dynamic gains in Example 2.

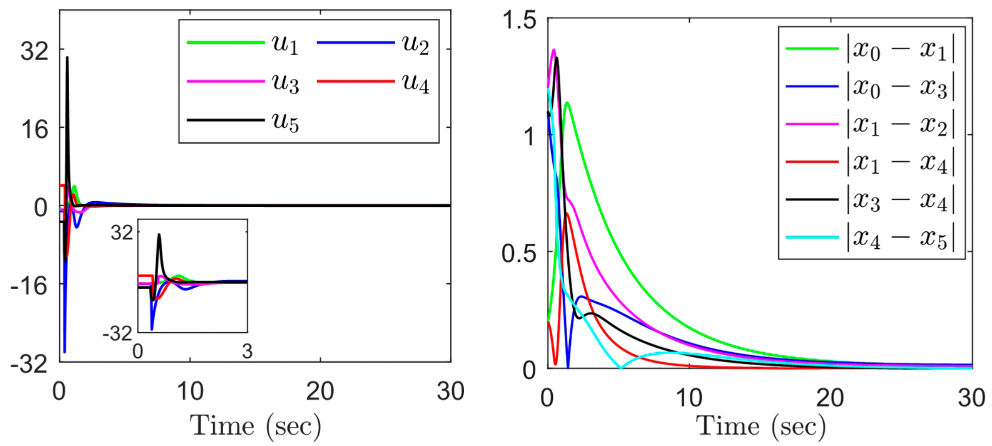

FIGURE 11 Evolution of the five followers' control protocols and the distances among initial neighboring leader and followers in Example 2.

$x_2(0), x_3(0), x_4(0), x_5(0)] = [0.6, 0.8, 2, -0.5, 0.6, -0.6]$, $[v_0(0), v_1(0), v_2(0), v_3(0), v_4(0)] = [0.1, 0.4, 1, 0.8, 0.5, 0.8]$, $[\hat{\rho}_1(0), \hat{\rho}_2(0), \hat{\rho}_3(0), \hat{\rho}_4(0), \hat{\rho}_5(0)] = [-0.5, 0.2, -0.6, -1, -3]$, $[\hat{\theta}_1(0), \hat{\theta}_2(0), \hat{\theta}_3(0), \hat{\theta}_4(0), \hat{\rho}_5(0)] = [-0.8, 2, -1, -0.2, -2]$. Then, by simulation, Figures 9–11 are given. From figures, we see that the five followers and the leader achieve consensus, the adaptive dynamic gains are bounded, the event-triggered protocol of each agent converges to zero and the distances between initial neighboring leader and followers are always less than $r = 3.5$. Hence, the proposed adaptive event-triggered connectivity-preserving consensus strategy is also effective for the leader-following scenario.

7 | CONCLUDING REMARKS

This paper has achieved the connectivity-preserving consensus via adaptive event-triggered control for the uncertain nonlinear MASs. Particularly, the investigated MASs admit the nonidentical unknown control coefficients and the heterogeneous nonlinearities with parameter uncertainties. With the help of potential functions and two dynamic gains, an adaptive event-triggered protocol has been developed for each agent to handle the severe system uncertainties and nonlinearities, the negative effect of the execution error and to achieve the consensus of MASs and the connectivity preservation of initial communication graph. Moreover, an extended study has been conducted on the leader-following scenario. Note that the signs of the control coefficients in this paper are known. One future work is to explore whether it is feasible to develop an adaptive event-triggered connectivity-preserving consensus strategy when the control coefficients have the unknown nonidentical signs. In addition, with the proposal of optimal control strategies (see e.g., References 42,49–52) which balance desired performance and available control resources, another future work is to seek an connectivity-preserving optimal control strategy for nonlinear MASs with unknown control coefficients.

ACKNOWLEDGMENTS

This work was supported in part by National Natural Science Foundation of China under grant/award 62033007 and 61821004, in part by Taishan Scholars Climbing Program of Shandong Province, in part by Major Fundamental Research Program of Shandong Province.

CONFLICT OF INTEREST STATEMENT

The authors declare no potential conflict of interests.

DATA AVAILABILITY STATEMENT

Data sharing not applicable to the paper as no datasets were generated or analyzed during the current study.

ORCID

Yungang Liu https://orcid.org/0000-0002-4753-9578

REFERENCES

1. Jadbabaie A, Lin J, Morse AS. Coordination of groups of mobile autonomous agents using nearest neighbor rules. *IEEE Trans Automat Contr*. 2003;48(6):988-1001.
2. Olfati-Saber R, Murray RM. Consensus problems in networks of agents with switching topology and time-delays. *IEEE Trans Automat Contr*. 2004;49(9):1520-1533.
3. Xie GM, Wang L. Consensus control for a class of networks of dynamic agents. *Int J Robust Nonlinear Control*. 2007;17(10-11):941-959.
4. Ren W. On consensus algorithms for double-integrator dynamics. *IEEE Trans Automat Contr*. 2008;53(6):1503-1509.
5. Das A, Lewis FL. Distributed adaptive control for synchronization of unknown nonlinear networked systems. *Automatica*. 2010;46(12):2014-2021.
6. Li ZK, Ren W, Liu XD, Fu MY. Consensus of multi-agent systems with general linear and Lipschitz nonlinear dynamics using distributed adaptive protocols. *IEEE Trans Automat Contr*. 2013;58(7):1786-1791.
7. Peng JM, Ye XD. Cooperative control of multiple heterogeneous agents with unknown high-frequency-gain signs. *Syst Control Lett*. 2014;68:51-56.
8. Niu XL, Liu YG, Man YC. Distributed adaptive consensus of nonlinear multi-agent systems with unknown control coefficients. *IFAC-PapersOnLine*. 2015;48(28):915-920.
9. Niu XL, Liu YG, Man YC. Consensus via time-varying feedback for uncertain nonlinear multi-agent systems with rather coarse input disturbances. *Syst Control Lett*. 2017;105:70-77.
10. Huang JS, Wang W, Wen CY, Zhou J, Li GQ. Distributed adaptive leader-follower and leaderless consensus control of a class of strict-feedback nonlinear systems: a unified approach. *Automatica*. 2020;118:109021.
11. Wang CL, Wen CY, Guo L. Adaptive consensus control for nonlinear multiagent systems with unknown control directions and time-varying actuator faults. *IEEE Trans Automat Contr*. 2021;66(9):4222-4229.
12. Ma X, Yang L, Ma L, et al. Distributed neuroadaptive monitoring fault-tolerant consensus control for nonlinear multi-agent systems with actuator faults. *Int J Robust Nonlinear Control*. 2022;32(9):5350-5378.
13. Razafindralambo T, Simplot-Ryl D. Connectivity preservation and coverage schemes for wireless sensor networks. *IEEE Trans Automat Contr*. 2011;56(10):2418-2428.
14. Beard RW, Lawton J, Hadaegh FY. A coordination architecture for spacecraft formation control. *IEEE Trans Control Syst Technol*. 2001;9(6):777-790.
15. Ji M, Egerstedt M. Distributed coordination control of multiagent systems while preserving connectedness. *IEEE Trans Robot*. 2007;23(4):693-703.
16. Liu ZX, Guo L. Synchronization of multi-agent systems without connectivity assumptions. *Automatica*. 2009;45:2744-2753.
17. Zavlanos MM, Tanner HG, Jadbabaie A, Pappas GJ. Hybrid control for connectivity preserving flocking. *IEEE Trans Automat Contr*. 2009;54(12):2869-2875.
18. Su HS, Chen GR, Wang XF, Lin ZL. Adaptive second-order consensus of networked mobile agents with nonlinear dynamics. *Automatica*. 2011;47(2):368-375.
19. Wang MM, Su HS, Zhao MM, Chen MZQ, Wang HW. Flocking of multiple autonomous agents with preserved network connectivity and heterogeneous nonlinear dynamics. *Neurocomputing*. 2013;115:169-177.

20. Dong Y, Huang J. Flocking with connectivity preservation of multiple double integrator systems subject to external disturbances by a distributed control law. *Automatica.* 2015;55:197-203.
21. Zhang Q, Li P, Yang ZQ, Chen ZQ. Adaptive flocking of nonlinear multi-agents systems with uncertain parameters. *IET Control Theory Appl*. 2015;9(3):351-357.
22. Ai XD, Song SJ, You KY. Second-order consensus of multi-agent systems under limited interaction ranges. *Automatica.* 2016;68: 329-333.
23. Yoo SJ. Connectivity-preserving consensus tracking of uncertain nonlinear strict-feedback multiagent systems: an error transformation approach. *IEEE Trans Neural Netw Learn Syst*. 2017;29(9):4542-4548.
24. Dong Y, Xu SY. A novel connectivity-preserving control design for rendezvous problem of networked uncertain nonlinear systems. *IEEE Trans Neural Netw Learn Syst*. 2020;31(12):5127-5137.
25. Restrepo E, Loría A, Sarras I, Marzat J. Edge-based strict Lyapunov functions for consensus with connectivity preservation over directed graphs. *Automatica.* 2021;132:109812.
26. He SD, Dai SL, Zhao ZJ, Zou T, Ma YF. UDE-based distributed formation control for MSVs with collision avoidance and connectivity preservation. *IEEE Trans Industr Inform*. doi:10.1109/TII.2023.3274234
27. Sandee JH, Visser PM, Heemels WPMH. Analysis and experimental validation of processor load for event-driven controllers. Proceedings of the 2006 IEEE International Conference on Control Application, Munich, Germany. 2006.
28. Lehmann D, Lunze J. Extension and experimental evaluation of an event-based state-feedback approach. *Control Eng Pract*. 2011;19(2):101-112.
29. Dimarogonas DV, Frazzoli E, Johansson KH. Distributed event-triggered control for multi-agent systems. *IEEE Trans Automat Contr*. 2012;57(5):1291-1297.
30. Seyboth GS, Dimarogonas DV, Johansson KH. Event-based broadcasting for multi-agent average consensus. *Automatica.* 2013;49(1):245-252.
31. Li HQ, Liao XF, Huang TW, Zhu W. Event-triggering sampling based leader-following consensus in second-order multi-agent systems. *IEEE Trans Automat Contr*. 2014;60(7):1998-2003.
32. Zhu W, Jiang ZP, Feng G. Event-based consensus of multi-agent systems with general linear models. *Automatica.* 2014;50(2): 552-558.
33. Xing LT, Wen CY, Guo FH, Liu ZT, Su HY. Event-based consensus for linear multiagent systems without continuous communication. *IEEE Trans Cybern*. 2016;47(8):2132-2142.
34. Wang W, Wen CY, Huang JS, Zhou J. Adaptive consensus of uncertain nonlinear systems with event triggered communication and intermittent actuator faults. *Automatica.* 2020;111:108667.
35. Yu H, Antsaklis PJ. Formation control of multi-agent systems with connectivity preservation by using both event-driven and time-driven communication. Proceedings of the 51st IEEE Conference on Decision and Control, Maui, Hawaii, USA. 2012.
36. Yu P, Ding L, Liu ZW, Guan ZH. Leader-follower flocking based on distributed event-triggered hybrid control. *Int J Robust Nonlinear Control.* 2016;26(1):143-153.
37. Yi XL, Wei JQ, Dimarogonas DV, Johansson KH. Formation control for multi-agent systems with connectivity preservation and event-triggered controllers. *IFAC-PapersOnline*. 2017;50(1):9367-9373.
38. Fan Y, Chen J, Song C, Wang Y. Event-triggered coordination control for multi-agent systems with connectivity preservation. *Int J Control Autom Syst*. 2020;18(4):966-979.
39. Zou Y, An Q, Miao SX, Chen SM, Wang XM, Su HS. Flocking of uncertain nonlinear multi-agent systems via distributed adaptive event-triggered control. *Neurocomputing.* 2021;465:503-513.
40. Dong Y, Xu SY. Rendezvous with connectivity preservation problem of linear multiagent systems via parallel event-triggered control strategies. *IEEE Trans Cybern*. 2022;52(5):2725-2734.
41. Wu X, Dong Y. An internal-model-based event-triggered strategy for rendezvous with connectivity preservation problem of multi-agent systems. *Int J Robust Nonlinear Control*. 2022;32(16):8874-8888. doi:10.1002/rnc.6298
42. Tong SC, Sun KK, Sui S. Observer-based adaptive fuzzy decentralized optimal control design for strict-feedback nonlinear large-scale system. *IEEE Trans Fuzzy Syst*. 2018;26(2):569-584.
43. Xie DS, Xu SY, Zhang BY, Li YM, Chu YM. Consensus for multi-agent systems with distributed adaptive control and an event-triggered communication strategy. *IET Control Theory Appl*. 2016;10(13):1547-1555.
44. Qian YY, Wan Y. Design of distributed adaptive event-triggered consensus control strategies with positive minimum inter-event times. *Automatica.* 2021;133:109837.
45. Shi YX, Hu QL. Event-driven connectivity-preserving coordinated control for multiple spacecraft systems with a distance-dependent dynamic graph. *IEEE Trans Cybern*. 2021;52(11):12551-12560.
46. Hao Y, Song S, Huang B, Li J. Distributed edge-event triggered formation control for multiple unmanned surface vessels with connectivity preservation. Proceedings of the 17th International Conference on Control & Automation, Naples, Italy. 2022.
47. Hale JK. *Ordinary Differential Equations*. 2nd ed. Krieger; 1980.
48. Min YY, Liu YG. Barbalat lemma and its application in analysis of system stability (in Chinese). *J Shandong Univ (Eng Sci)*. 2007;37(1):51-55.
49. Vamvoudakis KG, Lewis FL. Online actor-critic algorithm to solve the continuous-time infinite horizon optimal control problem. *Automatica*. 2010;46:878-888.

50. Ning B, Han QL, Zuo ZY. Distributed optimization of multiagent systems with preserved network connectivity. *IEEE Trans Cybern*. 2019;49(11):3980-3990.
51. Li YM, Min X, Tong SC. Adaptive fuzzy inverse optimal control for uncertain strict-feedback nonlinear systems. *IEEE Trans Fuzzy Syst*. 2020;28(10):2363-2374.
52. Li YM, Liu YJ, Tong SC. Observer-based neuro-adaptive optimized control for a class of strict-feedback nonlinear systems with state constraints. *IEEE Trans Fuzzy Syst*. 2022;33(7):3131-3145.

How to cite this article: Guo T, Liu Y. Connectivity-preserving consensus: An adaptive event-triggered strategy. *Int J Robust Nonlinear Control*. 2023;1-17. doi: 10.1002/rnc.7169