Received: 30 March 2023 | Revised: 22 July 2023 | Accepted: 19 October 2023

DOI: 10.1002/rnc.7072

RESEARCH ARTICLE

WILEY

Adaptive event-triggered control for nonlinear systems with time-varying parameter uncertainties

Lei Chu | **Yungang Liu**

School of Control Science and Engineering, Shandong University, Jinan, China

Correspondence
Yungang Liu, School of Control Science and Engineering, Shandong University, Jinan 250061, China.
Email: lygfr@sdu.edu.cn

Funding information
National Natural Science Foundation of China, Grant/Award Numbers: 62033007, 61821004; Taishan Scholars Climbing Program of Shandong Province; Major Fundamental Research Program of Shandong Province

Abstract

The paper considers adaptive event-triggered control for nonlinear systems with time-varying parameter uncertainties. The time-varying uncertainties do not need to be differentiable, but their variation amplitudes are required to be known. Such uncertainties appear not only in the system nonlinearities but also as the control coefficient. We pursue a tight design scheme by combining tuning functions and the congelation of variables method, thereby avoiding the overuse of dominations and obtaining a less conservative controller. In view of the essence of the control problem, we don't design a continuous controller first and then search for a complex and suitable event-triggering mechanism, as emulation methods done in the literature. In contrast, we adopt a simple event-triggering mechanism with the relative threshold, while seeking a complex and appropriate adaptive controller which contains the new adaptive treatment for execution error due to the unknown time-varying control coefficient. Note that there are three parameter dynamic compensators in the adaptive controller, and one is specialized to the execution error, which is not required in the continuous feedback context. It is shown that with the proposed adaptive event-triggered controller, all the closed-loop signals are bounded, the system state converges to zero, and no Zeno behavior occurs. A comparative simulation is provided to verify the theoretical results.

KEYWORDS

adaptive event-triggered control, nonlinear system, time-varying uncertainties

1 | INTRODUCTION AND PROBLEM FORMULATION

Adaptive event-triggered control is promising because of its potential in resource efficiency and feedback capabilities.[1-5] Although much progress has been made, adaptive event-triggered control still needs in-depth research. For example, integrated compensation mechanisms are expected for various uncertainties and various nonlinearities in the discontinuous sampling context. Execution errors, for some typical cases such as unknown time-varying control coefficients, call for new adaptive treatment. The tight design schemes of event-triggered controller should be pursued to avoid the overuse of dominations and in turn to reduce conservatism.

In this paper, we consider adaptive event-triggered control of the following nonlinear systems with time-varying parameter uncertainties:

$$\begin{cases} \dot{x}_i = x_{i+1} + \theta_i^{\mathrm{T}}(t)\phi_i(x_{[i]}), \; i = 1, \ldots, n-1, \\ \dot{x}_n = b(t)u + \theta_n^{\mathrm{T}}(t)\phi_n(x), \end{cases} \tag{1}$$

where $x=[x_1,\ \dots\ ,x_n]^{\mathrm{T}}\in\mathbf{R}^n$ is the system state with $x(\mathbf{0})=x_0$, $x_{[i]}=[x_1,\ \dots\ ,x_i]^{\mathrm{T}}$; $u\in\mathbf{R}$ is the control input; $\phi_i:\mathbf{R}^i\rightarrow\mathbf{R}^{q_i}$, $i=1,\ \dots\ ,n$ are smooth nonlinear functions with $\phi_i(\mathbf{0})=\mathbf{0}$. The vectors $\theta_i(t)\in\mathbf{R}^{q_i}$, $i=1,\ \dots\ ,n$ and control coefficient $b(t)\in\mathbf{R}\setminus\{0\}$ are unknown time-varying parameters. In addition, $\theta_i(t)$, $i=1,\ \dots\ ,n$ and $b(t)$ are (piecewise) continuous.

System (1) (as well as their variants) is a representative form of various strict-feedback nonlinear systems, which has been extensively studied during the past few decades.[6-8] Diverse practical plants can be modeled as the form of system (1), such as the mass-spring systems subject to time-varying external forces,[9] the servo motor system[10,11] and the induction motor system.[12] Inspired by the related results,[8,13] we impose the following assumptions on time-varying uncertainties to ensure the feasibility of adaptive event-triggered control.

Assumption 1. The parameters $\theta_i(t)$, $i=1,\ \dots\ ,n$ belong to an unknown compact set Θ, but the "radius" of Θ, denoted by δ_{Δ_θ}, is assumed to be known.

Assumption 2. The control coefficient $b(t)\in B$, where B is an unknown compact set and its "radius," denoted by δ_{Δ_b}, is known. Also, the sign of $b(t)$ is known and unchanged.

In Assumption 1, the known "radius" δ_{Δ_θ} implies that the variation amplitude of time-varying parameter $\theta(t)$ is known, where $\theta(t)=\left[\theta_1^{\mathrm{T}}(t),\theta_2^{\mathrm{T}}(t),\ \dots\ ,\theta_n^{\mathrm{T}}(t)\right]^{\mathrm{T}}$. To be specific, see from Assumption 1 that there are two unknown constant vectors $\overline{\theta}$ and $\underline{\theta}$ with the appropriate dimensions such that $\underline{\theta}\leq\theta(t)\leq\overline{\theta}$, for any $t\geq 0$, where the notation "$\leq$" is defined element-wise. Then, the "radius" of Θ refers to $\delta_{\Delta_\theta}=\frac{1}{2}\|\overline{\theta}-\underline{\theta}\|$, which is assumed to be known. A larger value of δ_{Δ_θ} means a larger uncertainty system (1) allows. In fact, such constraint (i.e., known "radius") is reasonable since it can easily be satisfied in real life. For example, in motor vehicle or aircraft systems, the variation amplitudes between the maximum and minimum safe driving speeds over any interval are usually known. Notably, even if δ_{Δ_θ} is *unknown*, we can also handle it by building an "estimate" for it using classical adaptive technique. But we will not discuss the problem here because it can't provide any significant improvement and overcomplicates our problem.

The "radius" δ_{Δ_b} in Assumption 2 has a similar definition to the "radius" δ_{Δ_θ}. Moreover, from Assumption 2, we can find a constant ℓ_b such that $\mathrm{sign}(\ell_b)=\mathrm{sign}(b(t))$ and $0<|\ell_b|\leq|b(t)|$. Without loss of generality, we assume the sign of $b(t)$ is positive. Then, the deviation of the control coefficient, defined as $\Delta_b=b(t)-\ell_b$, is positive and bounded by δ_{Δ_b}. It is worth pointing out that if the control direction, that is, the sign of $b(t)$, is unknown, compensating $b(t)$ is not challenging by employing Nussbaum-type function (see Reference 14). We thus do not pursue this in the paper.

Notably, unknown parameters $\theta_i(t)$, $i=1,\ \dots\ ,n$ and $b(t)$ of system (1), as distinct from those in References 15-17, are time-varying and allowed to be nondifferentiable. Thus, these parameters cannot appear in Lyapunov function candidate, making it challenging for conventional adaptive techniques to directly "estimate" them. One method to compensate time-varying parameters is so-called projection operation,[18-20] which confines parameter dynamic compensators within a prespecified compacted set to guarantee the boundedness of them. This method can achieve asymptotic stabilization/tracking only if the derivatives of time-varying parameters are $\mathscr{L}_1$. Another approach is switching σ-modification,[21-23] which adds leakages to parameter update laws if parameter estimates drift out of a prespecified region to ensure the boundedness of parameter estimates. However, this approach cannot achieve asymptotic stabilization/tracking when unknown parameters are persistently varying. Therefore, the above two methods are no longer valid for asymptotically stabilizing system (1), where unknown parameters are allowed to be nondifferentiable and persistently varying.

The robust adaptive control can achieve asymptotic stabilization of system (1), such as adaptive sliding-mode-like control[24] and continuous robust adaptive control.[25] However, such control offen suffers from the overuse of dominations and noise amplification problems, leading to conservative design. Recently, work[8] proposed a less conservative control scheme based on the congelation of variables method, where a tight controller was obtained by constraining the variation amplitudes of the time-varying uncertainties as in Assumptions 1 and 2. However, the above result do not take event-triggered communication into account, which motivates the research of this paper.

The objective of this paper is to pursue a tight adaptive event-triggered stabilizing controller for system (1) under Assumptions 1 and 2. The main characteristic of the tight controller is that the conservative upper bounds of time-varying uncertainties are not estimated and utilized throughout the controller design process, thus avoiding the overuse of dominations and reducing conservatism.

In fact, pursuing a tight design scheme for the nonlinear systems with time-varying parameter uncertainties would bring a series of difficulties and complexities, especially in the event-triggered setting. This is partially because the tight design framework imposes limitations on the utilization of upper or lower bounds of time-varying uncertainties. As a result, the execution error and nonlinearities cannot be compensated by virtual controllers that contain parameter compensators designed for the bounds of uncertainties, as in References 5,26, and 27. Recently, work[8] introduced the congelation of variables method to compensate the time-varying uncertainties, and in turn the nonlinearities can be counteracted via a tight controller. However, the compensation mechanism in work[8] is nontrivial to incorporate into the event-triggering framework because the execution error interacting with the control coefficient cannot be suppressed. This compels us to seek for a powerful design scheme containing a new adaptive treatment to counteract both the nonlinearities and execution error.

In this paper, we first introduce a new time-varying parameter vector. It can integrate unknown vectors $\theta_i(t)$, $i = 1, \ldots, n$, thereby transforming system (1) into another system with only one parameter vector. This allows us to dominate parameters $\theta_i(t)$, $i = 1, \ldots, n$ with just one parameter dynamic compensator by resorting to tuning functions. Then, a distinct Lyapunov function, motivated by the congelation of variables method, is constructed, in which the time-varying uncertainties themselves and their bounds are not involved. Particularly, a simple event-triggered mechanism with the relative threshold is introduced to save communication resources and channel bandwidth; however, the resulting execution error, due to multiplication by a time-varying and potentially nondifferentiable coefficient $b(t)$, becomes more challenging to counteract. To this end, we specifically introduce a parameter dynamic compensator and an important decomposition (i.e., (31)) to deal with the negative effects caused by coefficient $b(t)$, and then suppress the execution error by working with an appropriate controller. Moreover, another parameter dynamic compensator and a new decomposition (i.e., (32)) are given to handle the nonlinearities affected by coefficient $b(t)$. Based on this, a tight controller is derived by backstepping approach such that, for any initial value, all the closed-loop system signals are bounded, and system state x converges to zero while Zeno behavior does not occur.

By comparing this paper and existing results,[6,7,10,28-30] the main contributions of the proposed scheme are listed as follows: (i) Time-varying parameter uncertainties in the systems under investigation are allowed to be nondifferentiable and persistently varying. But results[10,29,30] require the time-varying uncertainties to be continuously differentiable and their first derivatives to be bounded. Importantly, when the uncertainties persistently vary, the control schemes[10,29,30] can only achieve certain boundedness rather than convergence. (ii) A tight design scheme is pursued. We build a tight controller by avoiding the application of conservative upper bounds of time-varying uncertainties. However, in works,[6,7,28] upper and lower bounds of time-varying uncertainties are always used during the controller design, which leads to the overuse of dominations and conservative design. Moreover, under the tight design framework, to compensate the nonlinearities and especially execution error in the presence of unknown time-varying uncertainties, we develop a complex and appropriate adaptive controller. The controller contains three new parameter dynamic compensators, and one is specialized to the execution error, which is not necessary in the continuous feedback context.

2 | ADAPTIVE EVENT-TRIGGERED CONTROLLER DESIGN

This section aims to construct an adaptive event-triggered stabilizing controller for system (1) under Assumptions 1 and 2. Notably, in controller design, we avoid estimating and utilizing the conservative upper bounds of time-varying uncertainties. Thus, it is possible to get a tight controller.

To facilitate suppressing unknown parameters $\theta_i(t)$, $i = 1, \ldots, n$, we lump them into vector $\theta(t)$, that is, $\theta(t) = \left[\theta_1^{\mathrm{T}}(t), \theta_2^{\mathrm{T}}(t), \ldots, \theta_n^{\mathrm{T}}(t)\right]^{\mathrm{T}} \in \mathbf{R}^q$ with $q = \sum_{j=1}^{n} q_j$. Also, we let $\overline{\phi}_i(x_{[i]}) = \left[\mathbf{0}_{q_1}^{\mathrm{T}}, \ldots, \mathbf{0}_{q_{i-1}}^{\mathrm{T}}, \phi_i^{\mathrm{T}}(x_{[i]}), \mathbf{0}_{q_{i+1}}^{\mathrm{T}}, \ldots, \mathbf{0}_{q_n}^{\mathrm{T}}\right]_{q\times 1}^{\mathrm{T}}$, where $\mathbf{0}_{q_i}$ denotes the q_i-dimensional zero vector. Then, system (1) can be rewritten as:

$$\begin{cases} \dot{x}_i = x_{i+1} + \theta^{\mathrm{T}}(t)\overline{\phi}_i(x_{[i]}),\ i = 1, \ldots, n-1, \\ \dot{x}_n = b(t)u + \theta^{\mathrm{T}}(t)\overline{\phi}_n(x). \end{cases} \tag{2}$$

For system (2), we define the following coordinate transformation:

$$\begin{cases} z_1 = x_1, \\ z_i = x_i - \alpha_{i-1}(x_{[i-1]}, \hat{\theta}, \hat{b}),\ i = 2, \ldots, n, \end{cases} \tag{3}$$

where smooth functions $\alpha_i(\cdot)$, $i = 1, \dots, n-1$ are designed as follows:

$$\begin{aligned}\alpha_i(x_{[i]}, \hat{\theta}, \hat{b}) = & -z_{i-1} - \left(c_i + \frac{n-i+1}{2}\delta_{\Delta_\theta}^2 + \frac{\left|\overline{\omega}_i(x_{[i]}, \hat{\theta}, \hat{b})\right|_{\mathrm{F}}^2}{2} + \frac{1}{2} + \frac{\delta}{2}\left(\hat{b} + \delta_{\Delta_b}\right) \right) z_i - \omega_i^{\mathrm{T}}(x_{[i]}, \hat{\theta}, \hat{b})\hat{\theta} \\ & + \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} x_{j+1} + \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \sum_{j=1}^{i} \omega_j(x_{[j]}, \hat{\theta}, \hat{b}) z_j + \sum_{j=2}^{i-1} \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \omega_i(x_{[i]}, \hat{\theta}, \hat{b}) z_j \\ & + \frac{\partial \alpha_{i-1}}{\partial \hat{b}} \sum_{j=1}^{i} \frac{\delta}{2} z_j^2 + \sum_{j=2}^{i-1} \frac{\partial \alpha_{j-1}}{\partial \hat{b}} \frac{\delta}{2} z_i z_j, \end{aligned} \tag{4}$$

with $z_0 = 0$, $\alpha_0(\cdot) = 0$ and $|\overline{\omega}_i(\cdot)|_{\mathrm{F}} = \sqrt{\sum_{k=1}^{i}\sum_{j=1}^{q}(\overline{\omega}_{i,kj})^2}$.

In (4), c_i, $i = 1, \dots, n$ are arbitrarily chosen positive constants introduced to provide stabilizing terms in Lyapunov analysis and then ensure the stabilization of the controlled system; $\hat{\theta}$ is the dynamic estimate of ℓ_θ that is often regarded as the average of $\theta(t)$; and design functions $\omega_i(\cdot)$, $i = 1, \dots, n$ are defined as:

$$\omega_i(x_{[i]}, \hat{\theta}, \hat{b}) = \overline{\phi}_i(x_{[i]}) - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} \overline{\phi}_j(x_{[j]}). \tag{5}$$

In (4), $\overline{\omega}_i(\cdot)$, $i = 1, \dots, n$ are smooth functions and satisfy $\omega_i(\cdot) = \overline{\omega}_i(\cdot) z_{[i]}$. Specifically, according to the smoothness of $\omega_i(x_{[i]}, \hat{\theta}, \hat{b})$ and $\omega_i(\mathbf{0}, \hat{\theta}, \hat{b}) = \mathbf{0}$, it follows that $\omega_i(x_{[i]}, \hat{\theta}, \hat{b}) = W_i(x_{[i]}, \hat{\theta}, \hat{b}) x_{[i]}$ with matrix-valued smooth function $W_i(\cdot)$. Also, since functions $\alpha_i(x_{[i]}, \hat{\theta}, \hat{b})$, $i = 1, \dots, n$ are smooth, we yield from (3) and (5) that $x_{[i]} = \overline{W}_i(x_{[i]}, \hat{\theta}, \hat{b}) z_{[i]}$ with matrix-valued smooth function $\overline{W}_i(\cdot)$. Thus, we can express $\omega_i(\cdot)$ as $\omega_i(\cdot) = \overline{\omega}_i(\cdot) z_{[i]}$, where $\overline{\omega}_i(\cdot) = W_i(\cdot)\overline{W}_i(\cdot)$.

We design the tight adaptive event-triggered controller as:

$$\begin{cases} u(t) = \alpha_n\left(x(t_k), \hat{\theta}(t_k), \hat{b}(t_k), \hat{\rho}(t_k)\right), \ \forall t \in [t_k, t_{k+1}), \\ \alpha_n(x, \hat{\theta}, \hat{b}, \hat{\rho}) = -\frac{\hat{\rho}}{2}\|\overline{\psi}\|^2 z_n - \frac{\delta}{2}\left(\|\overline{W}_n\|^2 + 1\right) z_n, \\ \dot{\hat{\theta}} = \sum_{j=1}^{n} \omega_j z_j, \ \hat{\theta}(0) \in \mathbf{R}^q, \\ \dot{\hat{b}} = \frac{\delta}{2}\sum_{j=1}^{n-1} z_j^2, \ \hat{b}(0) \in \mathbf{R}, \\ \dot{\hat{\rho}} = \frac{1}{2}\|\overline{\psi}\|^2 z_n^2, \ \hat{\rho}(0) \in \mathbf{R}^+, \end{cases} \tag{6}$$

where $\hat{b}$ is the dynamic estimate of ℓ_b, $\hat{\rho}$ is the dynamic estimate of $\frac{1}{\ell_b}$, $\overline{\psi}$ is a to-be-determined smooth function, and execution times t_k's are determined by the following event-triggering mechanism ($t_1 = 0$):

$$t_{k+1} = \inf\left\{ t > t_k \middle| \left| u(t) - \alpha_n\left(x(t), \hat{\theta}(t), \hat{b}(t), \hat{\rho}(t)\right) \right| > \delta\|x\| \right\}. \tag{7}$$

In (7), threshold parameter δ is a prespecified arbitrary positive constant, which is chosen by the designer according to hardware devices and task requirements. Notably, the smaller the δ, the faster the convergence speed, but the more frequent sampling/execution and communication.

Remark 1. We develop two parameter dynamic compensators (i.e., $\hat{b}$ and $\hat{\rho}$) to counteract the negative effects of time-varying control coefficient $b(t)$. Compensator $\hat{b}$ is specifically designed to deal with the coefficient $b(t)$ that multiplies by execution error, which is not needed in the continuous feedback context. On the other hand, compensator $\hat{\rho}$ is introduced to overcome the coefficient $b(t)$ that multiplies by the nonlinearities containing z_n in step n. Notably, such coefficient $b(t)$ cannot be compensated by compensator $\hat{b}$ because it is not allowed to contain z_n. To be specific, compensator $\hat{b}$ needs to enter the virtual controllers α_i, $i = 1, \dots, n-1$ to offset the unexpected terms caused by the execution error. Consequently, if $\hat{b}$ contains z_n, the partial derivatives of α_i, $i = 1, \dots, n-1$ would yield new nonlinearities concerning z_n. Such nonlinearities cannot be suppressed by controller due to the presence of the control coefficient $b(t)$, and furthermore, they would make the recursive design into a dilemma. Therefore, we introduce compensator $\hat{\rho}$, which exists only in α_n. This is also the reason why virtual controllers α_i, $i = 1, \dots, n-1$ and α_n are different in form.

Next, we give an important lemma that provides a delicate characterization for the dynamic behavior of the closed-loop system via Lyapunov function candidate. Particularly, we utilize unknown constant vector ℓ_θ and constant ℓ_b in Lyapunov function candidate rather than time-varying parameters themselves (i.e., $\theta(t)$ and $b(t)$), which avoids producing uncertain (or non-existent) terms $\dot{\theta}(t)$ and $\dot{b}(t)$ when taking the derivative of Lyapunov function candidate. This process is called *congelation of variables*.

Lemma 1. *For system* (2) *under Assumptions 1 and 2, the event-triggered controller* (6) *with triggering mechanism* (7) *makes Lyapunov function*

$$V = \frac{1}{2}\sum_{i=1}^{n} z_i^2 + \frac{1}{2}(\ell_\theta - \hat{\theta})^{\mathrm{T}}(\ell_\theta - \hat{\theta}) + \frac{1}{2}(\ell_b - \hat{b})^2 + \frac{1}{2}\ell_b\left(\frac{1}{\ell_b} - \hat{\rho}\right)^2, \tag{8}$$

satisfy that, on $[t_k, t_{k+1})$,

$$\dot{V} \le -\sum_{i=1}^{n} c_i z_i^2, \tag{9}$$

where constant vector ℓ_θ *belongs to* Θ *and can be regarded as the average of* $\theta(t)$; *constant* $\ell_b > 0$ *is the lower bound of* $b(t)$.

Proof of Lemma 1. We proceed with a recursive manner. To obtain a tight controller, we do not use conservative upper bounds of time-varying parameters throughout the controller design process, and furthermore, only use one parameter dynamic compensator to deal with all unknown parameters $\theta_i(t), i = 1, \dots, n$ by restoring to tuning functions.

Step 1. Let $V_1 = \frac{1}{2}z_1^2 + \frac{1}{2}(\ell_\theta - \hat{\theta})^{\mathrm{T}}(\ell_\theta - \hat{\theta})$. Then, taking the time derivative of V_1 along the trajectories of system (2) and applying transformation (3), we have

$$\dot{V}_1 = z_1(\theta^{\mathrm{T}}\overline{\phi}_1 + z_2 + \alpha_1) - (\ell_\theta - \hat{\theta})^{\mathrm{T}}\dot{\hat{\theta}}. \tag{10}$$

By adding and subtracting two terms $z_1\hat{\theta}^{\mathrm{T}}\omega_1$ and $z_1\ell_\theta^{\mathrm{T}}\omega_1$, we yield from the definition of ω_1 in (5) that

$$z_1\theta^{\mathrm{T}}\overline{\phi}_1 = z_1\theta^{\mathrm{T}}\omega_1 = z_1\hat{\theta}^{\mathrm{T}}\omega_1 + z_1\Delta_\theta^{\mathrm{T}}\omega_1 + z_1(\ell_\theta - \hat{\theta})^{\mathrm{T}}\omega_1, \tag{11}$$

where $\Delta_\theta(t) = \theta(t) - \ell_\theta$ indicates the deviation of $\theta(t)$, and $\|\Delta_\theta(t)\| \le \delta_{\Delta_\theta}$.

We next estimate the second term on the right-hand side of (11). From the definition of $\overline{\omega}_1$ after (5), there is

$$z_1\Delta_\theta^{\mathrm{T}}\omega_1 = z_1\Delta_\theta^{\mathrm{T}}\overline{\omega}_1 z_1 \le \frac{1}{2}\|\Delta_\theta\|^2 z_1^2 + \frac{1}{2}\|\overline{\omega}_1\|^2 z_1^2 \le \frac{1}{2}\delta_{\Delta_\theta}^2 z_1^2 + \frac{1}{2}|\overline{\omega}_1|_{\mathrm{F}}^2 z_1^2, \tag{12}$$

where δ_{Δ_θ} and $|\overline{\omega}_1|_{\mathrm{F}}$ have been defined in Assumption 1 and (4), respectively.

We define the first tuning function as:

$$\tau_1 = \omega_1 z_1. \tag{13}$$

Then, substituting (11) and (12) into (10), and by virtual control law α_1 in (4), we conclude

$$\dot{V}_1 \le z_1 z_2 - c_1 z_1^2 - \frac{n-1}{2}\delta_{\Delta_\theta}^2 z_1^2 - \frac{1}{2}\left(1 + \delta\left(\hat{b} + \delta_{\Delta_b}\right)\right)z_1^2 + (\ell_\theta - \hat{\theta})^{\mathrm{T}}(\tau_1 - \dot{\hat{\theta}}). \tag{14}$$

Recursive design step *i*(*i* = 2, …, *n* − 1). Suppose that steps $2, \dots, i-1$ have been completed, and V_{i-1} satisfies

$$\begin{aligned}\dot{V}_{i-1} \le{}& z_{i-1}z_i - \sum_{j=1}^{i-1} c_j z_j^2 - \frac{n-i+1}{2}\delta_{\Delta_\theta}^2 \sum_{j=1}^{i-1} z_j^2 - \sum_{j=1}^{i-1}\frac{1}{2}\left(1 + \delta\left(\hat{b} + \delta_{\Delta_b}\right)\right)z_j^2 \\ &- \sum_{j=2}^{i-1}\frac{\partial\alpha_{j-1}}{\partial\hat{\theta}}\sum_{k=i}^{n}\omega_k z_k z_j - \sum_{j=2}^{i-1}\frac{\partial\alpha_{j-1}}{\partial\hat{b}}\sum_{k=i}^{n-1}\frac{\delta}{2}z_k^2 z_j + (\ell_\theta - \hat{\theta})^{\mathrm{T}}(\tau_{i-1} - \dot{\hat{\theta}}),\end{aligned} \tag{15}$$

where tuning functions $\tau_i = \tau_{i-1} + \omega_i z_i, i = 2, \dots, n-2$.

Choose $V_i = V_{i-1} + \frac{1}{2}z_i^2$ for step i. By (2) and (3), there is

$$\dot{V}_i = \dot{V}_{i-1} + z_i(\theta^{\mathrm{T}}\overline{\phi}_i + z_{i+1} + \alpha_i - \dot{\alpha}_{i-1}). \tag{16}$$

Noting (4) and (6), we have

$$z_i\dot{\alpha}_{i-1} = z_i\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}(x_{j+1} + \theta^{\mathrm{T}}\overline{\phi}_j) + z_i\frac{\partial\alpha_{i-1}}{\partial\hat{\theta}}\sum_{j=1}^{n}\omega_j z_j + z_i\frac{\partial\alpha_{i-1}}{\partial\hat{b}}\frac{\delta}{2}\sum_{j=1}^{n-1}z_j^2. \tag{17}$$

By the definition of ω_i in (5) and the treatment similar to (11), we deduce

$$z_i\theta^{\mathrm{T}}\overline{\phi}_i - z_i\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\theta^{\mathrm{T}}\overline{\phi}_j = z_i\theta^{\mathrm{T}}\omega_i = z_i\hat{\theta}^{\mathrm{T}}\omega_i + z_i\Delta_\theta^{\mathrm{T}}\omega_i + z_i(\ell_\theta - \hat{\theta})^{\mathrm{T}}\omega_i. \tag{18}$$

From Assumption 1 and the definition of $\overline{\omega}_i$ after (5), it follows that

$$\begin{aligned}
z_i\Delta_\theta^{\mathrm{T}}\omega_i &= z_i\Delta_\theta^{\mathrm{T}}\overline{\omega}_i z_{[i]} = z_i\begin{bmatrix}\Delta_{\theta 1} & \Delta_{\theta 2} & \cdots & \Delta_{\theta q}\end{bmatrix}\cdot\begin{bmatrix}(\overline{\omega}_i)_{11} & (\overline{\omega}_i)_{12} & \cdots & (\overline{\omega}_i)_{1i}\\ (\overline{\omega}_i)_{21} & (\overline{\omega}_i)_{22} & \cdots & (\overline{\omega}_i)_{2i}\\ \vdots & \vdots & \ddots & \vdots\\ (\overline{\omega}_i)_{q1} & (\overline{\omega}_i)_{q2} & \cdots & (\overline{\omega}_i)_{qi}\end{bmatrix}\cdot\begin{bmatrix}z_1\\ z_2\\ \vdots\\ z_i\end{bmatrix}\\
&= z_i\Delta_{\theta 1}\sum_{k=1}^{q}(\overline{\omega}_i)_{k1}z_1 + \cdots + z_i\Delta_{\theta q}\sum_{k=1}^{q}(\overline{\omega}_i)_{ki}z_i\\
&\le \frac{1}{2}\|\Delta_\theta\|^2 z_1^2 + \frac{1}{2}\sum_{k=1}^{q}\|\overline{\omega}_i\|_{k1}^2 z_i^2 + \cdots + \frac{1}{2}\|\Delta_\theta\|^2 z_i^2 + \frac{1}{2}\sum_{k=1}^{q}\|\overline{\omega}_i\|_{ki}^2 z_i^2\\
&\le \sum_{j=1}^{i}\frac{1}{2}\delta_{\Delta_\theta}^2 z_j^2 + \frac{1}{2}|\overline{\omega}_i|_{\mathrm{F}}^2 z_i^2.
\end{aligned} \tag{19}$$

We select the ith tuning function as:

$$\tau_i = \tau_{i-1} + \omega_i z_i. \tag{20}$$

Then, substituting (17)–(19) into (16) and invoking (4) and (15) yield

$$\begin{aligned}
\dot{V}_i \le{}& z_i z_{i+1} - \sum_{j=1}^{i}c_j z_j^2 - \frac{n-i}{2}\delta_{\Delta_\theta}^2\sum_{j=1}^{i}z_j^2 - \sum_{j=1}^{i}\frac{1}{2}\left(1 + \delta\left(\hat{b} + \delta_{\Delta_b}\right)\right)z_j^2\\
&- \sum_{j=2}^{i}\frac{\partial\alpha_{j-1}}{\partial\hat{\theta}}\sum_{k=i+1}^{n}\omega_k z_k z_j - \sum_{j=2}^{i}\frac{\partial\alpha_{j-1}}{\partial\hat{b}}\sum_{k=i+1}^{n-1}\frac{\delta}{2}z_k^2 z_j + (\ell_\theta - \hat{\theta})^{\mathrm{T}}(\tau_i - \dot{\hat{\theta}}).
\end{aligned} \tag{21}$$

This completes the first $n-1$ steps. In step n, the analysis is quite different from that in previous steps due to unknown time-varying control coefficient $b(t)$ and execution error $|u(t) - \alpha_n(t)|$, and we therefore discuss it in detail. Importantly, a new adaptive treatment for the execution error is offered in this step, which is one of the main contributions of the paper.

Step *n*. We let $V = V_{n-1} + \frac{1}{2}z_n^2 + \frac{1}{2}(\ell_b - \hat{b})^2 + \frac{1}{2}\ell_b\left(\frac{1}{\ell_b} - \hat{\rho}\right)^2$ for this step. Then, by (2) and (3), we have

$$\dot{V} = \dot{V}_{n-1} + z_n(\theta^{\mathrm{T}}\overline{\phi}_n + bu - \dot{\alpha}_{n-1}) - (\ell_b - \hat{b})\dot{\hat{b}} - \ell_b\left(\frac{1}{\ell_b} - \hat{\rho}\right)\dot{\hat{\rho}}. \tag{22}$$

From (4) and (6), it follows that

$$z_n\dot{\alpha}_{n-1} = z_n\sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}(x_{j+1}+\theta^{\mathrm{T}}\overline{\phi}_j) + z_n\frac{\partial\alpha_{n-1}}{\partial\hat{\theta}}\sum_{j=1}^{n}\omega_j z_j + z_n\frac{\partial\alpha_{n-1}}{\partial\hat{b}}\frac{\delta}{2}\sum_{j=1}^{n-1}z_j^2. \tag{23}$$

By the definition of ω_n in (5), we have

$$z_n\theta^{\mathrm{T}}\overline{\phi}_n - z_n\sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}\theta^{\mathrm{T}}\overline{\phi}_j = z_n\theta^{\mathrm{T}}\omega_n = z_n\hat{\theta}^{\mathrm{T}}\omega_n + z_n\Delta_\theta^{\mathrm{T}}\omega_n + z_n(\ell_\theta - \hat{\theta})^{\mathrm{T}}\omega_n. \tag{24}$$

Similar to (19), there is

$$z_n\Delta_\theta^{\mathrm{T}}\omega_n \le \sum_{j=1}^{n}\frac{1}{2}\delta_{\Delta_\theta}^2 z_j^2 + \frac{1}{2}|\overline{\omega}_n|_{\mathrm{F}}^2 z_n^2. \tag{25}$$

Substituting (23)–(25) into (22), by (21), we obtain

$$\begin{aligned}\dot{V} \le{}& z_{n-1}z_n - \sum_{j=1}^{n-1}c_j z_j^2 - \frac{1}{2}\delta_{\Delta_\theta}^2\sum_{j=1}^{n-1}z_j^2 - \sum_{j=1}^{n-1}\frac{1}{2}\left(1+\delta\left(\hat{b}+\delta_{\Delta_b}\right)\right)z_j^2 - \sum_{j=2}^{n-1}\frac{\partial\alpha_{j-1}}{\partial\hat{\theta}}\omega_n z_n z_j\\ &+ (\ell_\theta - \hat{\theta})^{\mathrm{T}}(\tau_{n-1} - \dot{\hat{\theta}}) + z_n\hat{\theta}^{\mathrm{T}}\omega_n + \frac{1}{2}\sum_{j=1}^{n}\delta_{\Delta_\theta}^2 z_j^2 + \frac{1}{2}|\overline{\omega}_n|_{\mathrm{F}}^2 z_n^2 + (\ell_\theta - \hat{\theta})^{\mathrm{T}}z_n\omega_n + buz_n\\ &- z_n\sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}x_{j+1} - z_n\frac{\partial\alpha_{n-1}}{\partial\hat{\theta}}\sum_{j=1}^{n}\omega_j z_j - z_n\frac{\partial\alpha_{n-1}}{\partial\hat{b}}\frac{\delta}{2}\sum_{j=1}^{n-1}z_j^2 - (\ell_b - \hat{b})\dot{\hat{b}} - \ell_b\left(\frac{1}{\ell_b} - \hat{\rho}\right)\dot{\hat{\rho}}.\end{aligned}$$

By defining $\tau_n = \tau_{n-1} + \omega_n z_n$, the above inequality can be derived as:

$$\begin{aligned}\dot{V} \le{}& buz_n - \sum_{j=1}^{n-1}c_j z_j^2 - \sum_{j=1}^{n-1}\frac{1}{2}\left(1+\delta\left(\hat{b}+\delta_{\Delta_b}\right)\right)z_j^2 + (\ell_\theta - \hat{\theta})^{\mathrm{T}}(\tau_n - \dot{\hat{\theta}}) - (\ell_b - \hat{b})\dot{\hat{b}} - \ell_b\left(\frac{1}{\ell_b} - \hat{\rho}\right)\dot{\hat{\rho}}\\ &+ z_n\left(z_{n-1} - \sum_{j=2}^{n-1}\frac{\partial\alpha_{j-1}}{\partial\hat{\theta}}\omega_n z_j + \hat{\theta}^{\mathrm{T}}\omega_n + \frac{1}{2}\delta_{\Delta_\theta}^2 z_n + \frac{1}{2}|\overline{\omega}_n|_{\mathrm{F}}^2 z_n - \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}x_{j+1} - \frac{\partial\alpha_{n-1}}{\partial\hat{\theta}}\sum_{j=1}^{n}\omega_j z_j - \frac{\partial\alpha_{n-1}}{\partial\hat{b}}\frac{\delta}{2}\sum_{j=1}^{n-1}z_j^2\right).\end{aligned} \tag{26}$$

Invoking the dynamics of $\hat{\theta}$ in (6) and the definition of τ_n yields

$$\dot{V} \le b\alpha_n z_n + b(u-\alpha_n)z_n - \sum_{j=1}^{n}c_j z_j^2 - \sum_{j=1}^{n-1}\frac{1}{2}\left(1+\delta\left(\hat{b}+\delta_{\Delta_b}\right)\right)z_j^2 - \frac{1}{2}z_n^2 + z_n\psi - (\ell_b - \hat{b})\dot{\hat{b}} - \ell_b\left(\frac{1}{\ell_b} - \hat{\rho}\right)\dot{\hat{\rho}}, \tag{27}$$

where $\psi(x,\hat{\theta}) = \frac{1}{2}z_n + c_n z_n + z_{n-1} - \sum_{j=2}^{n-1}\frac{\partial\alpha_{j-1}}{\partial\hat{\theta}}\omega_n z_j + \hat{\theta}^{\mathrm{T}}\omega_n + \frac{1}{2}\delta_{\Delta_\theta}^2 z_n + \frac{1}{2}|\overline{\omega}_n|_{\mathrm{F}}^2 z_n - \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}x_{j+1} - \frac{\partial\alpha_{n-1}}{\partial\hat{\theta}}\sum_{j=1}^{n}\omega_j z_j - \frac{\partial\alpha_{n-1}}{\partial\hat{b}}\frac{\delta}{2}\sum_{j=1}^{n-1}z_j^2$.

Note that $\psi(x,\hat{\theta})$ is smooth and $\psi(\mathbf{0},\hat{\theta}) = \mathbf{0}$. Then, similar to the definition of $\overline{\omega}$ after (5), we get $\psi(x,\hat{\theta}) = \overline{\psi}^{\mathrm{T}}(x,\hat{\theta})z$ with $\overline{\psi}$ smooth function. Based on this, we estimate the sixth term on the right-hand side of (27) as follows:

$$z_n\psi = z_n\overline{\psi}^{\mathrm{T}}z \le \sum_{j=1}^{n}\frac{1}{2}z_j^2 + \frac{1}{2}\|\overline{\psi}\|^2 z_n^2. \tag{28}$$

We next calculate the second term on the right-hand side of (27), which refers to the execution error term. By the definition of $\overline{W}_n$ after (5) and triggering mechanism (7), we have, on $[t_k, t_{k+1})$,

$$b(u-\alpha_n)z_n \le b\delta\|x\|\cdot|z_n| = b\delta\|\overline{W}_n z\|\cdot|z_n| \le \frac{b\delta}{2}\|\overline{W}_n\|^2 z_n^2 + \frac{b\delta}{2}\sum_{j=1}^{n}z_j^2 = \frac{b\delta}{2}\left(\|\overline{W}_n\|^2+1\right)z_n^2 + \frac{b\delta}{2}\sum_{j=1}^{n-1}z_j^2. \tag{29}$$

Substituting (28) and (29) into (27) yields, on $[t_k, t_{k+1})$,

$$\dot{V} \le -\sum_{j=1}^{n} c_j z_j^2 + b\alpha_n z_n + \frac{b\delta}{2}\left(\|\overline{W}_n\|^2 + 1\right) z_n^2 + \frac{b\delta}{2}\sum_{j=1}^{n-1} z_j^2 - \sum_{j=1}^{n-1}\frac{\delta}{2}\left(\hat{b} + \delta_{\Delta_b}\right) z_j^2 + \frac{1}{2}\|\overline{\psi}\|^2 z_n^2 - (\ell_b - \hat{b})\dot{\hat{b}} - \ell_b\left(\frac{1}{\ell_b} - \hat{\rho}\right)\dot{\hat{\rho}}. \tag{30}$$

Notably, the fourth term on the right-hand side of (30), caused by the execution error (see (29)), cannot be suppressed by controller $\alpha_n(\cdot)$ because it is not multiplied by z_n. Furthermore, virtual controllers $\alpha_i(\cdot)$, $i = 1, \dots, n-1$ are unable to offset it directly because it contains unknown time-varying parameter $b(t)$. To this end, we do the following important decomposition:

$$\frac{b\delta}{2}\sum_{j=1}^{n-1} z_j^2 = \frac{\delta}{2}\hat{b}\sum_{j=1}^{n-1} z_j^2 + \frac{\delta}{2}\Delta_b\sum_{j=1}^{n-1} z_j^2 + \frac{\delta}{2}(\ell_b - \hat{b})\sum_{j=1}^{n-1} z_j^2, \tag{31}$$

where $0 < \Delta_b = b - \ell_b \le \delta_{\Delta_b}$ (see Assumption 2 and its interpretation). Then, the first two terms on the right-hand side of (31) can be offset by virtual controllers α_i, $i = 1, \dots, n-1$, where two competent damping terms have been pre-arranged. The last term of (31) can be counteracted by the dynamics of $\hat{b}$, which is specialized to "$\frac{b\delta}{2}\sum_{j=1}^{n-1} z_j^2$" and is not needed in the continuous feedback context.

The sixth term on the right-hand side of (30), caused by the system nonlinearities, also cannot be directly counteracted by the controller due to the existence of unknown control coefficient $b(t)$. Moreover, compensator $\hat{b}$ is unable to dominate the term because $\hat{b}$ is not allowed to contain z_n. We therefore resort to the compensator $\hat{\rho}$ and do another delicate decomposition:

$$\begin{aligned}\frac{1}{2}\|\overline{\psi}\|^2 z_n^2 &= \ell_b\left(\frac{1}{\ell_b} - \hat{\rho}\right)\frac{1}{2}\|\overline{\psi}\|^2 z_n^2 + \ell_b\hat{\rho}\frac{1}{2}\|\overline{\psi}\|^2 z_n^2 \\ &= \ell_b\left(\frac{1}{\ell_b} - \hat{\rho}\right)\frac{1}{2}\|\overline{\psi}\|^2 z_n^2 + b\hat{\rho}\frac{1}{2}\|\overline{\psi}\|^2 z_n^2 - \Delta_b\hat{\rho}\frac{1}{2}\|\overline{\psi}\|^2 z_n^2.\end{aligned} \tag{32}$$

Then, the first term of (32) can be dominated by the dynamics of $\hat{\rho}$, the second term can be offset by the virtual controller α_n and the perturbation term "$-\Delta_b\hat{\rho}\frac{1}{2}\|\overline{\psi}\|^2 z_n^2$" is nonpositive.

Substituting (31) and (32) into (30), and invoking α_n, $\dot{\hat{b}}$ and $\dot{\hat{\rho}}$ in (6), we obtain, on $[t_k, t_{k+1})$,

$$\dot{V} \le -\sum_{j=1}^{n} c_j z_j^2 - \Delta_b\hat{\rho}\frac{1}{2}\|\overline{\psi}\|^2 z_n^2. \tag{33}$$

By $\dot{\hat{\rho}} = \frac{1}{2}\|\overline{\psi}\|^2 z_n^2$ in (6) and $\hat{\rho}(0) > 0$, we know that $\hat{\rho}(t) > 0$ on the maximal existence interval of the solution of the closed-loop system. Thus, by $\Delta_b > 0$ in (31), we conclude that $-\Delta_b\hat{\rho}\frac{1}{2}\|\overline{\psi}\|^2 z_n^2 \le 0$ and (9) holds. ■

Remark 2. To deal with the control coefficient $b(t)$, we also use the congelation of variables method motivated by work,[8] but the analysis pattern is quite different from that of $\theta(t)$. But in work,[8] control coefficient $b(t)$ was treated in almost the same way as parameter $\theta(t)$, and the term $b(t)u(t)$ was sorted out into three terms including $\bar{u}$, $\frac{1}{\hat{\ell}_b}\Delta_b\bar{u}$ and $-\ell_b\left(\frac{1}{\ell_b} - \frac{1}{\hat{\ell}_b}\right)\bar{u}$, where $u = \frac{1}{\hat{\ell}_b}\bar{u}$. This seems unwise in the event-triggered context because such treatment would derive three intractable terms containing execution error. In contrast, we do not sort out $b(t)u(t)$ but exploit two dynamic parameter compensators (i.e., $\hat{b}$ and $\hat{\rho}$) and two important decompositions (i.e., (29) and (32)) to eliminate the influence of control coefficient $b(t)$, and thus we do not cause additional execution error terms.

For better understanding, we provide the following adaptive event-triggered controller design algorithm to intuitively display the design procedure (Algorithm 1).

Algorithm 1. Adaptive event-triggered controller design

Input: State x of system (2), "radius" δ_{Δ_θ} and "radius" δ_{Δ_b} in Assumptions 1 and 2, respectively, design parameters c_i, $i = 1, \dots, n$ in virtual controllers (4), and the prespecified parameter δ in triggering mechanism (7).
Output: Triggering instants $t_k, k = 1, 2, \dots$ and controller $u(t)$.
Initialization: $i = 1, t_1 = 0$;
while $i \le n$ **do**
1: Set V_i: $V_1 = \frac{1}{2}z_1^2 + \frac{1}{2}(\ell_\theta - \hat{\theta})^{\mathrm{T}}(\ell_\theta - \hat{\theta})$, $V_j = V_{j-1} + \frac{1}{2}z_j^2$ $(j = 2, \dots, n-1)$, $V_n = V_{n-1} + \frac{1}{2}z_n^2 + \frac{1}{2}(\ell_b - \hat{b})^2 + \frac{1}{2}\ell_b\left(\frac{1}{\ell_b} - \hat{\rho}\right)^2$;
2: Take the time derivative of V_i along the trajectories of system (2);
3: Design appropriate virtual controller α_i to compensate/dominate the destabilizing terms in $\dot{V}_i$;
if $i = n$ **then**
Design the dynamics of parameter compensators $\hat{\theta}$, $\hat{b}$ and $\hat{\rho}$ to counteract the uncertainties and execution error;
while $t_k \le T_e$ **do**
if $t \ge t_k$ && $|u(t_k) - \alpha_n(x(t), \hat{\theta}(t), \hat{b}(t), \hat{\rho}(t))| \ge \delta\|x\|$ **then**
$t_{k+1} = t$,
$u(t) = \alpha_n(t_{k+1})$.
end if
end while
end if
4: $i = i + 1$.
end while
return $t_k, k = 1, 2, \dots$ and $u(t)$.

3 | MAIN RESULTS

This section collects the main results on the adaptive event-triggered controller designed above, and particularly analyzes the performance of the closed-loop system composed of (2) and (6) under the triggering mechanism (7).

Note from system (2) that $\overline{\phi}_i(x_{[i]})$'s $i = 1, \dots, n$ are smooth and then locally Lipschitz. Thus, the right-hand side of the closed-loop system is locally Lipschitz in $(x, \hat{\theta}, \hat{b}, \hat{\rho})$ and continuous in t. By the existence and uniqueness theorem and the continuation theorem, one can obtain that, for any initial value $\left(x(0), \hat{\theta}(0), \hat{b}(0), \hat{\rho}(0)\right) \in \mathbf{R}^n \times \mathbf{R}^q \times \mathbf{R} \times \mathbf{R}^+$, the closed-loop system has a unique solution $\left(x(t), \hat{\theta}(t), \hat{b}(t), \hat{\rho}(t)\right)$ on the maximal existence interval $[0, T_e)$, where $T_e = +\infty$ or $0 < T_e < +\infty$. Here, $0 < T_e < +\infty$ means finite escape time exists or Zeno behavior occurs.

Before embarking on the concluding theorem, we give an important proposition, which greatly facilitates the analysis of system performance later on. The rigorous proof of the proposition is somewhat involved and thus is postponed to the Appendix.

Proposition 1. *Suppose that all the closed-loop system signals are bounded on* $[0, T_e)$. *Then, there is no Zeno phenomenon.*

Now, we present the concluding theorem on adaptive event-triggered stabilization.

Theorem 1. *Consider system* (2) *under Assumptions 1 and 2. The proposed adaptive event-triggered controller* (6) *with* (7) *can guarantee that, for any initial value* $\left(x(0), \hat{\theta}(0), \hat{b}(0), \hat{\rho}(0)\right) \in \mathbf{R}^n \times \mathbf{R}^q \times \mathbf{R} \times \mathbf{R}^+$, *the unique solution of the closed-loop system is defined on* $[0, +\infty)$, *and the following properties are satisfied*: (*i*) *all the closed-loop system signals are bounded*; (*ii*) *no infinitely fast execution, including Zeno behavior, occurs*; (*iii*) *system state x asymptotically converges to zero.*

Proof of Theorem 1. By Lemma 1, we have $\dot{V}(t) \le -\sum_{i=1}^{n} c_i z_i^2(t)$ for any $t \in [t_k, t_{k+1})$, which implies $V(t) - V(t_k) \le -\sum_{i=1}^{n} c_i \int_{t_k}^{t} z_i^2(s)\mathrm{d}s$. Then, from the continuity of V, it can be recursively obtained that, on $[0, T_e)$,

$$V(t) - V(0) \leq -\sum_{i=1}^{n} c_i \int_0^t z_i^2(s)\mathrm{d}s \leq 0. \tag{34}$$

By this and the definition of V in (8), we get

$$\frac{1}{2}\sum_{i=1}^{n} z_i^2(t) + \frac{1}{2}(\ell_\theta - \hat{\theta}(t))^{\mathrm{T}}(\ell_\theta - \hat{\theta}(t)) + \frac{1}{2}(\ell_b - \hat{b}(t))^2 + \frac{1}{2}\ell_b\left(\frac{1}{\ell_b} - \hat{\rho}(t)\right)^2 \leq V(0) < +\infty.$$

This implies that signals z_i's, $\hat{\theta}$, $\hat{b}$ and $\hat{\rho}$ are bounded on $[0, T_e)$, which, together with (2)–(6), leads to the boundedness of x_i's, $\alpha_i(\cdot)$'s and u. Then, we can conclude from Proposition 1 that there is no Zeno phenomenon, and furthermore, $T_e = +\infty$. Therefore, it follows that all signals of the closed-loop system are bounded and no infinitely fast execution occurs on $[0, +\infty)$.

We next prove the convergence of x. By (34), and noting the boundedness of V on $[0, +\infty)$, we have $\int_0^{+\infty} \|z(t)\|^2 \mathrm{d}t \leq +\infty$, which, together with transformation (3) and the boundedness of x_i's, $\alpha_i(\cdot)$'s and $\hat{\theta}$, implies $\int_0^{+\infty} \|x(t)\|^2 \mathrm{d}t \leq +\infty$. We also obtain from the boundedness of x_i's, u, θ and b that $\dot{x}$ is bounded. Then, utilizing Barbălat's Lemma yields $\lim_{t\to+\infty} x(t) = 0$.

This completes the proof of Theorem 1. ∎

4 | SIMULATION RESULTS

In this section, we verify the effectiveness of the proposed adaptive event-triggered control scheme by a time-varying mass-spring mechanical system. In addition, we provide two comparative experiments. The first aims to demonstrate the less conservative of the designed tight controller. For this, we compare our control scheme with conventional adaptive ones[6,7,28] that used conservative bounds (i.e. (37)) of uncertainties during the controller design. The second focuses on comparing the control methods with and without the event-triggered control, showing that the event-triggered control method can largely save communication and computation resources.

Consider the following controlled mass-spring model[9]:

$$m\ddot{y} + F_f + F_{sp} = F, \tag{35}$$

where m is the mass of the slider; y is the displacement from a reference position; $F_f = c(1 + \sin t)v$ is a frictional resistance with the sliding velocity $v = \dot{y}$ and an unknown constant $c > 0$; $F_{sp} = ky$ is the restoring force of the spring with the unknown spring constant k; and $F = c(1 + 0.5 \sin t)u$ is a time-varying external force acting on the sliding.

We first show the convergence of $(y, \dot{y})$ to $(0, 0)$ via the proposed adaptive event-triggered controller. Let $x_1 = y$ and $x_2 = \dot{y}$. Then, the objective is transformed into regulating (x_1, x_2) to $(0, 0)$ for the following system:

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = \frac{c(1+0.5\sin t)}{m}u - \frac{k}{m}x_1 - \frac{c(1+\sin t)}{m}x_2. \end{cases} \tag{36}$$

Clearly, system (36) satisfies Assumptions 1 and 2 with $\theta_1(t) = 0$, $\theta_2(t) = [\theta_{21}, \theta_{22}]^{\mathrm{T}} = \left[\frac{k}{m}, \frac{c(1+\sin t)}{m}\right]^{\mathrm{T}}$, $\phi_1(\cdot) = 0$, $\phi_2(\cdot) = [-x_1, -x_2]^{\mathrm{T}}$ and $b(t) = \frac{c(1+0.5\sin t)}{m}$.

Select virtual controller α_1 in the form of (4) with $q = 3$ and event-triggered controller u in the form of (6) with (7). Following the recursive design procedure in Section 2, we can deduce design functions involved: $\overline{\phi}_1(\cdot) = \omega_1(\cdot) = \overline{\omega}_1(\cdot) = 0$, $\overline{\phi}_2(\cdot) = \omega_2(\cdot) = [0, -x_1, -x_2]^{\mathrm{T}}$, $\overline{\omega}_2(\cdot) = \begin{bmatrix} 0 & 0 \\ -1 & 0 \\ C & -1 \end{bmatrix}$, $\overline{W}_2(\cdot) = \begin{bmatrix} 1 & 0 \\ -C & 1 \end{bmatrix}$, $\psi(\cdot) = \left(1 - C^2\right) z_1 + \frac{\delta^2}{4}z_1^3 + \left(\frac{1}{2}\delta_{\Delta_\theta}^2 + \frac{1}{2}|\overline{\omega}_2|_{\mathrm{F}}^2 + C + \frac{1}{2} + c_2\right) z_2 + \hat{\theta}^{\mathrm{T}}\omega_2$ and $\overline{\psi}(\cdot) = \left[1 - C^2 + \frac{\delta^2}{4}z_1^2 - \hat{\theta}_2 + \hat{\theta}_3 C, \frac{1}{2}\delta_{\Delta_\theta}^2 + \frac{1}{2}|\overline{\omega}_2|_{\mathrm{F}}^2 + C + \frac{1}{2} + c_2 - \hat{\theta}_3\right]^{\mathrm{T}}$, where $C = c_1 + \delta_{\Delta_\theta}^2 + \frac{1}{2} + \frac{\delta}{2}\left(\hat{b} + \delta_{\Delta_b}\right)$ and $\hat{\theta}_i$ is the ith entry of vector $\hat{\theta}$, $i = 2, 3$.

Let $m = k = c = 1$, $c_1 = 1$, $c_2 = 2$, $\delta_{\Delta_\theta} = \delta_{\Delta_b} = 0.2$ and $\delta = 0.1$. With the initial values: $x_1(0) = -4$, $x_2(0) = 6$, $\hat{\theta}(0) = [0.6, 0.6, 0.6]^{\mathrm{T}}$, $\hat{b}(0) = 0.6$ and $\hat{r}(0) = 0.6$, simulation results are displayed in Figures 1 and 2. Apparently, the figures show

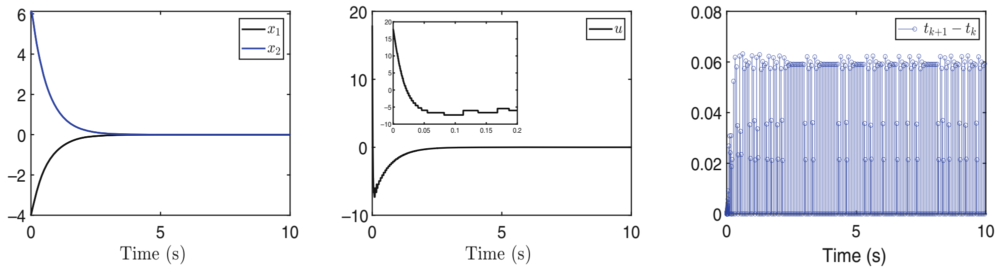

FIGURE 1 Evolution of system states x_1 and x_2, controller u and inter-execution time under our scheme.

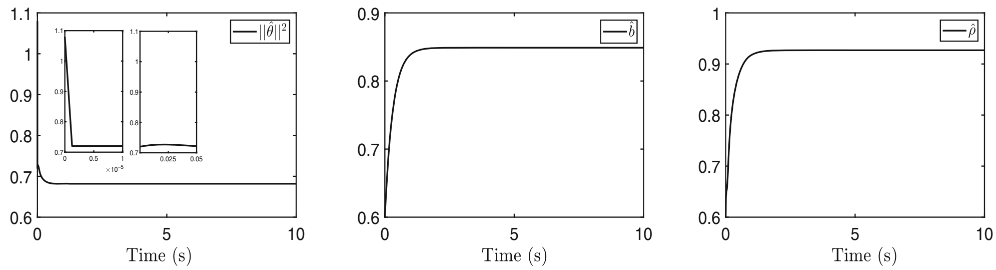

FIGURE 2 Evolution of $\|\hat{\theta}\|^2$, $\hat{b}$ and $\hat{\rho}$ under our scheme.

that all signals of the closed-loop system are bounded and system states (i.e., x_1 and x_2) converge to 0. In addition, Figure 1 illustrates that the proposed control scheme has no Zeno behavior.

To demonstrate the less conservative of the designed tight controller, we give a conventional adaptive event-triggered control scheme for system (36). Specifically, the coordinate transformation is the same as (3), and Lyapunov function candidate is selected as $V = \frac{1}{2}z_1^2 + \frac{1}{2\underline{b}}z_2^2 + \frac{1}{2}\tilde{r}^2$. In V, $\underline{b}$ is an unknown constant and satisfies $0 < \underline{b} \leq b(t)$; $\tilde{r} = r - \hat{r}$ and r is defined as:

$$r = \sup_{t\geq 0}\|\Omega(t)\|, \tag{37}$$

where $\Omega(t) = \left[\frac{\overline{\theta}}{\underline{b}}, \frac{\overline{\theta}}{\underline{b}}, \frac{D}{\underline{b}}, \frac{D^2}{\underline{b}}, \frac{\delta\overline{b}^2}{2\underline{b}^2}\right]$ with unknown constants $\overline{\theta}$ and $\overline{b}$ satisfying $\overline{\theta} = \sup_{t\geq 0}\|\theta(t)\|$ and $b(t) \leq \overline{b} < \infty$, respectively. To make V satisfy $\dot{V} \leq -c_1z_1^2 - c_2z_2^2$, the virtual controllers and parameter dynamic compensator are chosen as $\alpha_1(x_1, \hat{r}) = -Dz_1$, $\alpha_2(x, \hat{r}) = -c_2z_2 - z_2 - \frac{\delta}{2}z_2 - \frac{1}{4}\|\overline{\varphi}\|^2z_2 - \frac{1}{4}\hat{r}^2\|\overline{\varphi}\|^2z_2$ and $\dot{\hat{r}} = -\frac{1}{2}\|\overline{\varphi}\|^2z_2^2$, where $D = c_1 + 1 + \frac{\delta}{2}$, $\overline{\varphi} = \begin{bmatrix} -1 & 0 \\ c_1 & -1 \\ 0 & 1 \\ -1 & 0 \\ 0 & 2+D \end{bmatrix}$, δ is the prespecified threshold parameter in triggering mechanism, and c_1, c_2 are positive constants.

Choosing the same initial values as before, we get Figures 3 and 4, which show that the conventional control scheme can also achieve the asymptotical stabilization of system (36). Notably, by Figures 1 and 3, we can conclude that the control peak under our scheme is smaller than that under the conventional scheme. Therefore, we claim that the designed tight controller is less conservative in this point. Such less conservatism is because our scheme applies the congealed parameters of $\theta(t)$ and $b(t)$ in the controller design, rather than their bounds (i.e., (37)) like the conventional schemes. Hence, our control scheme avoids the overuse of dominations and leads to a less conservative controller.

We next compare the control methods with and without the event-triggered control. Removing the event-triggering mechanism from the proposed control scheme, we obtain a continuous-time control scheme. In particular, dynamic compensator $\hat{b}$ is no longer required since there is no execution error. Following the proposed scheme, the continuous-time controller is as follows: $\alpha_1(x_1, \hat{\theta}) = -C'z_1$ and $u(x, \hat{\theta}, \hat{\rho}) = -\frac{\hat{\rho}}{2}\|\overline{\psi}\|z_2$ with $C' = c_1 + \frac{1}{2}\delta_{\Delta_\theta} + \frac{1}{2}$, where the dynamics of $\hat{\theta}$ and $\hat{\rho}$, and smooth function $\overline{\psi}$ are the same as the proposed scheme. With the same parameters and initial values, the simulation results are shown in Figures 5 and 6.

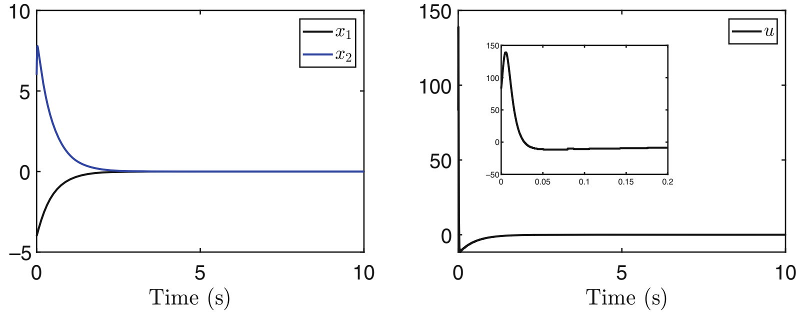

FIGURE 3 Evolution of system states x_1 and x_2 and controller u under the conventional adaptive control scheme.

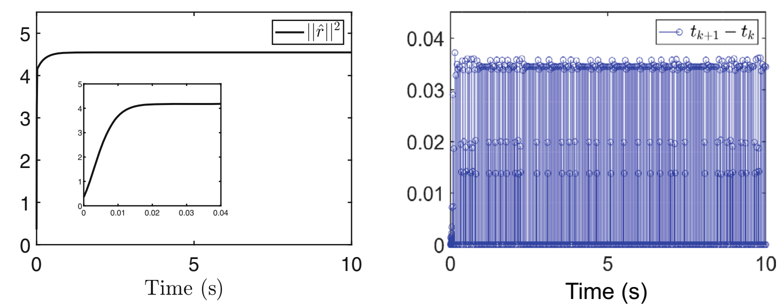

FIGURE 4 Evolution of $||\hat{r}||^2$ and inter-execution time under the conventional adaptive control scheme.

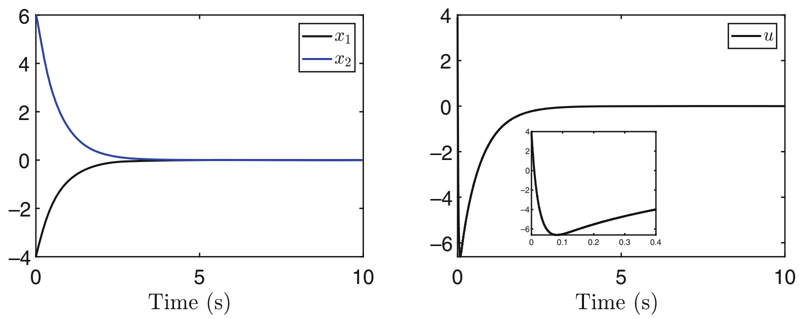

FIGURE 5 Evolution of system states x_1 and x_2 and controller u under the continuous-time control scheme.

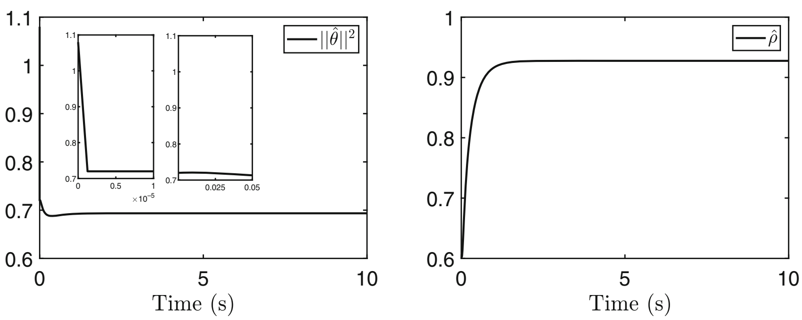

FIGURE 6 Evolution of $||\hat{\theta}||^2$ and $\hat{\rho}$ under the continuous-time control scheme.

By comparing the simulation results under the proposed scheme and the continuous-time control scheme, we find that the control peak in the latter is smaller than that in the former. This is reasonable because in continuous-time control scheme, richer and more complete information is transmitted and used for feedback, resulting in more timely and accurate control commands. However, the high performance of continuous-time control scheme comes at the cost of consuming a lot of communication/computation resources. With a step size of 1.25×10^{-6}, for example, the number of sampling during the initial 10 s of continuous-time control is infinitely large, while that of event-triggered control is only 409. Therefore, event-triggered control is superior to continuous-time control in terms of resource conservation.

5 | CONCLUDING REMARKS

In this paper, an adaptive event-triggered control scheme for global stabilization has been developed for nonlinear systems with time-varying parameter uncertainties. Particularly, we avoid taking the derivative of time-varying uncertainties and circumvent estimating/utilizing the conservative upper bounds of time-varying uncertainties throughout the controller design process. Then, the overuse of dominations is avoided and noise amplification problems are weakened. Thus, we claim that the designed controller is tight or less conservative. Nevertheless, the proposed scheme has some limitations. The scheme is tailored to the stabilization problem and cannot be directly extended to the tracking problem. Moreover, the scheme is restricted to the scenario where the information transmission between sensor and controller is continuous, and only transmission between controller and actuator is governed by an event-triggering mechanism. In the future, we will strive to extend the proposed scheme to a tracking control problem, while considering double-side (asynchronous) event-triggered control.

CONFLICT OF INTEREST STATEMENT

No conflicts of interest were declared by the authors in preparing this paper.

DATA AVAILABILITY STATEMENT

Data sharing not applicable to the paper as no datasets were generated or analyzed during the current study.

ORCID

Yungang Liu https://orcid.org/0000-0002-4753-9578

REFERENCES

1. Huang J, Wang W, Wen C, Li G. Adaptive event-triggered control of nonlinear systems with controller and parameter estimator triggering. *IEEE Trans Autom Control*. 2019;65(1):318-324.
2. Zhao F, Gao W, Liu T, Jiang Z. Adaptive optimal output regulation of linear discrete-time systems based on event-triggered output-feedback. *Automatica*. 2022;137:110103.
3. Li F, Liu Y. A new adaptive event-triggered scheme enabling inter-execution time pre-evaluation and double-side communication reduction. *IEEE Control Syst Lett*. 2022;6:2329-2334.
4. Xing L, Wen C, Liu Z, Su H, Cai J. Event-triggered adaptive control for a class of uncertain nonlinear systems. *IEEE Trans Autom Control*. 2016;62(4):2071-2076.
5. Huang Y, Liu Y. Practical tracking via adaptive event-triggered feedback for uncertain nonlinear systems. *IEEE Trans Autom Control*. 2019;64(9):3920-3927.
6. Ye X. Asymptotic regulation of time-varying uncertain nonlinear systems with unknown control directions. *Automatica*. 1999;35(5):929-935.
7. Li Y. Command filter adaptive asymptotic tracking of uncertain nonlinear systems with time-varying parameters and disturbances. *IEEE Trans Autom Control*. 2021;67(6):2973-2980.
8. Chen K, Astolfi A. Adaptive control for systems with time-varying parameters. *IEEE Trans Autom Control*. 2020;66(5):1986-2001.
9. Khalil H. *Nonlinear Systems*. 3rd ed. Prentice Hall; 2002.
10. Wu J, Zhao J, Wu D. Indirect adaptive robust control of nonlinear systems with time-varying parameters in a strict feedback form. *Int J Robust Nonlinear Control*. 2018;28(13):3835-3851.
11. Hong Y, Yao B. A globally stable saturated desired compensation adaptive robust control for linear motor systems with comparative experiments. *Automatica*. 2007;43:1840-1848.
12. Marino R, Peresada S, Tomei P. On-line stator and rotor resistance estimation for induction motors. *IEEE Trans Control Syst Technol*. 2000;8:570-579.

13. Chen K, Astolfi A. Adaptive control of linear systems with time-varying parameters. Paper presented at: Proceeding of the 2018 Annual American Control Conference (ACC). 2018; Milwaukee, WI:80-85.
14. Nussbaum R. Some remarks on a conjecture in parameter adaptive control. *Syst Control Lett*. 1983;3(5):243-246.
15. Krstic M, Kanellakopoulos I, Kokotovic P. Adaptive nonlinear control without overparametrization. *Syst Control Lett*. 1992;19(3):177-185.
16. Man Y, Liu Y. A powerful output-feedback controller for uncertain nonlinear systems. *Int J Robust Nonlinear Control*. 2018;28(6):2264-2275.
17. Li F, Liu Y. Global adaptive stabilization via asynchronous event-triggered output-feedback. *Automatica*. 2022;139:110181.
18. Goodwin G, Mayne D. A parameter estimation perspective of continuous time model reference adaptive control. *Automatica*. 1987;23(1):57-70.
19. Pomet J, Praly L. Adaptive nonlinear regulation: estimation from the Lyapunov equation. *IEEE Trans Autom Control*. 1992;37(6):729-740.
20. Patil O, Sun R, Bhasin S, Dixon W. Adaptive control of time-varying parameter systems with asymptotic tracking. *IEEE Trans Autom Control*. 2022;67(9):4809-4815.
21. Tsakalis K, Ioannou P. Adaptive control of linear time-varying plants. *Automatica*. 1987;23(4):459-468.
22. Zhang Y, Ioannou P. Adaptive control of linear time-varying systems. Paper presented at: Proceeding of the 35th IEEE Conference on Decision and Control (CDC). 1996; Kobe, Japan:837-842.
23. Zhang Y, Fidan B, Ioannou P. Backstepping control of linear time-varying systems with known and unknown parameters. *IEEE Trans Autom Control*. 2003;48(11):1908-1925.
24. Qu Z, Xu J. Model-based learning controls and their comparisons using Lyapunov direct method. *Asian J Control*. 2002;4(1):99-110.
25. Zhang Z, Xie X, Ge S. Adaptive tracking for uncertain MIMO nonlinear systems with time-varying parameters and bounded disturbance. *IEEE Trans Syst Man Cybern Syst*. 2019;51(7):4479-4491.
26. Li J, Liu C, Sun Y, Shao L. A new event-triggered adaptive tracking controller for nonlinear systems with unknown virtual control coefficients. *Eur J Control*. 2023;69:100759.
27. Ku C, Chang W, Lee T. Event-triggered control for Takagi-Sugeno fuzzy model-based nonlinear systems with time-varying parameters. *Int J Fuzzy Syst*. 2023;25:2152-2162. doi:10.1007/s40815-023-01495-0
28. Li T, Wen C, Yang J, Li S, Guo L. Event-triggered tracking control for nonlinear systems subject to time-varying external disturbances. *Automatica*. 2020;119:109070.
29. Marino R, Tomei P. Adaptive control of linear time-varying systems. *Automatica*. 2003;39(4):651-659.
30. Marino R, Tomei P. Robust adaptive regulation of linear time-varying systems. *IEEE Trans Autom Control*. 2000;45(7):1301-1311.

How to cite this article: Chu L, Liu Y. Adaptive event-triggered control for nonlinear systems with time-varying parameter uncertainties. *Int J Robust Nonlinear Control*. 2023;1-15. doi: 10.1002/rnc.7072

APPENDIX

Proof of Proposition 1. We consider the following two cases: (i) $\|x(t^*)\| = 0$, $\exists t^* \in [t_k, t_{k+1})$; (ii) $\|x(t)\| \neq 0$, $\forall t \in [t_k, t_{k+1})$.

In case (i), we have $x(t^*) = 0$, where $t^* \in [t_k, t_{k+1})$. Then, by (2)–(6), we get $u(t^*) = 0$ and $\overline{\phi}_i(x_{[i]}(t^*)) = 0$, and furthermore, $\dot{x}_i(t^*) = 0, i = 1, \ldots, n$. Thus, $\|x(t)\| = 0$ for any $t \geq t^*$, which, together with event-triggering mechanism (7), yields that the triggering condition is not satisfied any more and Zeno phenomenon does not occur.

In case (ii), we let $e(t) = |u(t) - \alpha_n(t)|$ and define $\eta(t) = \frac{e(t)}{\|x(t)\|}$ for $t \in [t_k, t_{k+1})$. Then, by the triggering mechanism (7), one can obtain that $\eta(t_k) = 0$ and $\eta\left(t_{k+1}^-\right) = \delta$.

Taking the time derivative of η, we have

$$D^+\eta = \frac{\|x\|D^+e - eD^+\|x\|}{\|x\|^2}. \tag{A1}$$

Note that $D^+\|x\| = \frac{x^{\mathrm{T}}}{\|x\|}D^+x \leq \|D^+x\|$. Then, from (2), there is

$$\begin{aligned}\|D^+x\| &\leq \sum_{i=1}^{n-1}\left|\overline{\phi}_i^{\mathrm{T}}\theta + x_{i+1}\right| + \left|\overline{\phi}_n^{\mathrm{T}}\theta + b\alpha_n + b(u-\alpha_n)\right| \\ &\leq \sum_{i=1}^{n}\left|\overline{\phi}_i^{\mathrm{T}}\theta\right| + \sum_{i=1}^{n}|x_i| + b|\alpha_n| + b|u-\alpha_n|.\end{aligned} \tag{A2}$$

By the method of completing square, we have

$$\begin{cases} \sum_{i=1}^{n}\left|\overline{\phi}_i^{\mathrm{T}}\theta\right| \leq \sqrt{n}\|\theta\|\cdot\|\overline{\phi}\| \leq \sqrt{n}\|\theta\|\cdot\|\Phi\|\cdot\|x\|, \\ \sum_{i=1}^{n}|x_i| \leq \sqrt{n}\|x\|, \end{cases} \tag{A3}$$

where $\overline{\phi} = \left[\overline{\phi}_1, \ldots, \overline{\phi}_n\right]^{\mathrm{T}}$ and $\Phi(x)$ is a smooth function. The existence of Φ is due to $\overline{\phi}(0) = 0$ and the smoothness of $\overline{\phi}(x)$.

Noting $z_n(0, \hat{\theta}, \hat{b}) = 0$ and the smoothness of $z_n(x, \hat{\theta}, \hat{b})$, we have $z_n = \Psi^{\mathrm{T}}(x, \hat{\theta}, \hat{b})x$, and then, by the definition of α_n in (6), we get

$$b|\alpha_n| \leq \frac{b}{2}\hat{\rho}\|\overline{\psi}\|^2\cdot|z_n| + \frac{\delta b}{2}\left(\|\overline{W}_n\|^2+1\right)\cdot|z_n| = \left(\frac{b}{2}\hat{\rho}\|\overline{\psi}\|^2 + \frac{\delta b}{2}\left(\|\overline{W}_n\|^2+1\right)\right)\cdot\|\Psi\|\cdot\|x\|, \tag{A4}$$

where $\Psi(x, \hat{\theta}, \hat{b})$ is a smooth function.

From the triggering mechanism, it follows that $|u - \alpha_n| \leq \delta\|x\|$ on $[t_k, t_{k+1})$. Then, substituting (A3) and (A4) into (A2) yields

$$\|D^+x\| \leq \beta_1(x, \hat{\theta}, \hat{b}, \hat{\rho})\|x\|, \tag{A5}$$

where $\beta_1(\cdot) = \sqrt{n}\|\theta\|\cdot\|\Phi\| + \sqrt{n} + \frac{b}{2}\hat{\rho}\|\overline{\psi}\|^2\cdot\|\Psi\| + \frac{\delta b}{2}\left(\|\overline{W}_n\|^2+1\right)\cdot\|\Psi\| + b\delta$.

Similar to (A5), by invoking (2) and (6), we have

$$\begin{aligned} D^+e = D^+|u-\alpha_n| &\leq |\dot{\alpha}_n| \\ &= \left|\sum_{i=1}^{n-1}\frac{\partial\alpha_n}{\partial x_i}(\overline{\phi}_i^{\mathrm{T}}\theta + x_{i+1}) + \frac{\partial\alpha_n}{\partial x_n}(\overline{\phi}_n^{\mathrm{T}}\theta + bu) + \frac{\partial\alpha_n}{\partial\hat{\theta}}\dot{\hat{\theta}} + \frac{\partial\alpha_n}{\partial\hat{b}}\dot{\hat{b}} + \frac{\partial\alpha_n}{\partial\hat{\rho}}\dot{\hat{\rho}}\right| \\ &\leq \left|\sum_{i=1}^{n}\frac{\partial\alpha_n}{\partial x_i}\overline{\phi}_i^{\mathrm{T}}\theta + \sum_{i=1}^{n-1}\frac{\partial\alpha_n}{\partial x_i}x_{i+1} + \frac{\partial\alpha_n}{\partial x_n}b\alpha_n + \frac{\partial\alpha_n}{\partial\hat{\theta}}\dot{\hat{\theta}} + \frac{\partial\alpha_n}{\partial\hat{b}}\dot{\hat{b}} + \frac{\partial\alpha_n}{\partial\hat{\rho}}\dot{\hat{\rho}}\right| + \left|\frac{\partial\alpha_n}{\partial x_n}b(u-\alpha_n)\right| \\ &\leq \beta_2(x, \hat{\theta}, \hat{b}, \hat{\rho})\|x\|, \end{aligned} \tag{A6}$$

where $\beta_2(\cdot)$ is a smooth function.

Substituting (A5) and (A6) into (A1), we obtain

$$D^+\eta \leq \frac{\beta_2\|x\|^2 + e\beta_1\|x\|}{\|x\|^2} = \beta_2 + \beta_1\eta \leq \lambda(\eta+1), \tag{A7}$$

where λ is the maximum of $\beta_1(\cdot)$ and $\beta_2(\cdot)$ on the maximal existence interval of the solution of the closed-loop system.

By the boundedness of x, $\hat{\theta}$, $\hat{b}$ and $\hat{\rho}$, we get that λ is bounded. Then, integrating both sides of (A7) yields $\eta(t) \leq e^{\lambda(t-t_k)} - 1$, which implies

$$t_{k+1} - t_k \geq \frac{1}{\lambda}\ln(\delta+1) > 0. \tag{A8}$$

Thus, Zeno behavior does not occur. ■